Eigenspaces of Graphs. *By Dragos Cvetkovic, Peter Rowlinson, and Slobodan Simic.* Cambridge University Press, Cambridge, UK, 1997. $69.95. xiii+258 pp., hardback. ISBN 0-521-57352-1.

This volume continues a series of monographs in algebraic graph theory that specialize to spectral graph theory — the study of interconnections between the properties of a graph and the eigenvalues and eigenvectors of the associated adjacency matrix. The common thread between the two previous works, *Spectra of Graphs — Theory and Applications* [3] and *Recent Results in the Theory of Graph Spectra* [2], and the volume being reviewed is the presence of Cvetkovic as one of the authors.

Spectra of Graphs is a self-contained introduction to the subject that includes references to all of the literature available through 1978. *Recent Results in the Theory of Graph Spectra* was written as an update, intending to survey all of the literature available through 1984. *Eigenspaces of Graphs* is different in two respects. While the previous volumes focused on eigenvalues, now, as the title indicates, the focus is on the associated eigenspaces. Second, rather than being an exhaustive survey of previous results the authors are more forward-looking, describing their current and on-going research into graph angles, star partitions and canonical star bases. The result is a text that is clearly written and instructive (though it is not a textbook) and which should be accessible to experienced researchers and graduate students.

In the early days of spectral graph theory it was hoped that knowledge of a graph's eigenvalues would determine the graph. Since the eigenvalues of a graph are independent of the labeling of the vertices it was hoped that the eigenvalues would allow one to easily determine if two graphs were isomorphic. However, the discovery of pairs of non-isomorphic graphs with identical spectra dashed these hopes. As computing power and techniques increased, many more such pairs were found and it became clear that even within restricted classes of graphs the spectra is far from sufficient for characterizing graphs. For example, Schwenk [4] showed that almost every tree has a cospectral mate.

Despite being insufficent as a characterization tool, eigenvalues are very helpful as a means of investigating a graph's properties. Translating a question about a graph into a question about its eigenvalues, and then applying techniques from linear algebra, is often a successful approach. Additionally, eigenvalues are useful in a variety of applications, most notably chemistry. It is easy to model a molecule with a graph. (Indeed, some of the early history of graph theory arose in this connection when Cayley related his results on trees to the problem of isomerism [1]). Then, using Hückel theory, the eigenvalues of the chemical graph provide predictions about various properties of the molecule, such as its aromaticity [5]. Applications to chemistry are some of the authors' main research interests, and the text and bibliography reflect this interest throughout.

In this volume, the authors turn their attention to the eigenspaces of a graph with the hope that this additional information will be enough to characterize graphs. However an eigenspace is a much more complicated object than an eigenvalue so the first problem is to determine a canonical description of an eigenspace. The construction of a star partition of the vertices is designed to overcome this hurdle and it leads to natural bases for the eigenspaces. Another algebraic invariant of a graph is the collection of angles of the eigenspaces and these are discussed thoroughly.

One cannot disassociate eigenvalues from eigenvectors, so the book begins with two chapters about previous results concerning both eigenvalues and eigenvectors and provides an excellent introduction to the current knowledge about these topics. The third chapter deals with the one-dimensional eigenspace associated with the largest eigenvalue of a connected graph (this eigenvalue is known as the index). The next three chapters continue to explore geometric properties of eigenspaces that are invariant of the labeling of a graph, such as the angles between eigenspaces.

Chapters 7 and 8 describe the authors' current research on star partitions and star bases. Here the motivation provided by the graph isomorphism problem intensifies. The final chapter collects some miscellaneous results and is followed by an appendix with a table of graph angles for every connected graph on 5 or fewer vertices. The over 300 references is a comprehensive, but not exhaustive list, of new and old works in spectral graph theory that includes a significant number of references to the literature in mathematical chemistry.

This book is a very well-written exposition of an active area of research in spectral graph theory and is recommended to anyone with an interest in algebraic graph theory. With the authors' long-standing interest in applications to chemistry it should also be of interest to those working in the intersection of mathematics and chemistry. Given its two introductory chapters, with frequent pointers to the existing literature, this is the kind of text that a graduate student could use to carry them from the basics of spectral graph theory to an understanding of a number of open questions.

References

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An edited version of this review was published in *SIAM Review* 40, No. 4 (1998) as part of the Book Review section.