Erdös on Graphs: His Legacy of Unsolved Problems. *By Fan Chung and Ron Graham.* A K Peters, Ltd., Wellesley, MA, 1998. $30.00. xii+142 pp., hardback. ISBN 1-56881-079-2.

This is an unusual book (in the best sense of the word) about an unusual man (again, in the best sense of the word). Paul Erdös was well known, and understood, within the worldwide community of mathematicians, but was a curiosity to those outside it. He was indefatigable, with more than 1486 publications to his credit [1]. Officially, he collaborated with at least 492 other mathematicians through joint publications [1]. And he made many important contributions to mathematics, often originating entire new fields of study. As a social person, his devotion to mathematics meant his life was most unique. Since his death in 1996 at the age of 83, two popular books about his life have appeared [3, 4]. While these are entertaining reading, the mathematics they contain leaves the professional mathematician wanting more. The book under review more than fills this gap.

Erdös was famous for being an itinerant mathematician — always traveling from one conference to another, or from one mathematics department to another, never staying anywhere more than a few weeks, but always doing mathematics. As he liked to say, "Another roof, another proof." However, in support of this lifestyle, the authors of this book attended to many of the more mundane details of his life. For example, they handled much of his professional correspondence and his bank account. They also devoted a room in their house as a sort of home base where Erdös could stay while visiting them. So as both friends and collaborators of Erdös (they have a total of 40 occurrences of joint authorship), they are as qualified as anyone to write about his life and his mathematics.

Erdös was known as a mathematician that loved to work on problems, rather than designing broad general theories. Though in so doing, he often created entire new branches of mathematics that led to more general theorems. He was accurately described as the "prince of problem solvers and the absolute monarch of problem posers" and it is this description that guides the structure of the book. Despite his contributions to many areas of combinatorics, number theory, probability and set theory, this book concentrates on his contributions to graph theory. It is organized into several chapters, each devoted to a specific topic in graph theory, starting with a description of the origin of the topic. It then goes on to describe, somewhat chronologically, the development of the topic through the problems posed and the problems solved. Of course, as the title of the book suggests, many of the problems posed have yet to be solved.

Each of these unsolved problems is set off from the text in a box, including details such as the proposers' names, and a reference, if available. As one flips through the book, almost every page seems to contain several boxed problems, and very few pages are without at least one. Erdös was fond of stimulating activity on certain problems by offering cash rewards for their solution, and indeed these rewards often served as a numerical measure of either the importance or perceived difficulty of a problem. Chung and Graham have decided to continue this tradition and will honor these offers, so some of the boxed problems contain a dollar figure in the upper right-hand corner.

What makes this book unusual is the mixture of accounts of Erdös' mathematics and accounts of Erdös as a person. The front matter contains a Preface, a two-page tribute titled "Remembering Uncle Paul," and an Introduction. In these places the authors convey their appreciation for Erdös and his work, while simultaneously giving some insight into Erdös' nature as a mathematician and a person. At the end of the book, there are twenty pages of "Erdös Stories" recounted by Andy Vázsonyi (who first collaborated with Erdös in 1936). Much of this material is well known, since many mathematicians have their own favorite Erdös story or saying (this reviewer's favorite is Erdös' penchant for stating one of his philosophies of life by saying that "Property is a nuisance."). While much of this material appears in the popular accounts of his life [2, 3, 4], it is refreshing to hear it from sources who knew the man, and his mathematics, so well.

Thoughtfully, the many references given are listed as footnotes. The Introduction also contains descriptions of four bibliographies of Erdös' publications and descriptions of nine survey papers on various aspects of his work. The mathematical chapters are titled: "Ramsey Theory," "Extremal Graph Theory," "Coloring, Packing and Covering," "Random Graphs and Graph Enumeration," "Hypergraphs," "Infinite Graphs."

Surveying the range of mathematics contained would be difficult in a review such as this. Given that Erdös was such a prolific practitioner of combinatorics, "encyclopedic" would almost be a proper description of an account of his lifetime of work. Instead we will mention his work on random graphs and give a sampling of the problems discussed.

The "probabilistic method" is a technique pioneered by Erdös and Rényi in 1959 that has been a very successful method for studying and enumerating graphs with specific properties. This is one example of an instance where his work has spawned an entire theory. Briefly, one associates a probability function with a set of graphs. This can be done in a variety of ways, for example, by considering the set of graphs on n vertices with m edges and giving each of the $\binom{\binom{n}{2}}{m}$ labeled graphs an equal probability. Or, one can assume that each possible edge of a graph on n vertices is included in the graph with probability p. The latter scenario is described by referring to a "random graph." The behavior of this set of graphs as p is varied can be quite surprising.

With this definition we can quote the following problems, which appear consecutively in the space of little more than a page:

- Let G be a random graph on n vertices with cn edges. What is the smallest c for which the probability that the chromatic number is r is at least some constant strictly greater than 0 (and independent of n)?

- How accurately can one estimate the chromatic number of a random graph (with edge probability $\frac{1}{2}$)? Prove or disprove that the range of expected values is more (much more) than $O(1)$.

- (with Bollobás) A random graph (with edge density $\frac{1}{2}$) on $n = 2^d$ vertices contains a d-cube.

- (with Bollobás) In a random graph (with edge probability $\frac{1}{2}$) find the best possible c such that every subgraph on n^α vertices will almost surely contain an independent set of size $c\log(n)$ (where c depends on α).

- (with Spencer) Start with n vertices and add edges at random one at a time. If we stop when every vertex is contained in a triangle, is there almost surely a set of vertex disjoint triangles covering every vertex (except for at most two vertices)?

This book will appeal to a variety of readers. For the professional mathematician, the vignettes on Erdös' life will be of as much interest as the recently published popular accounts and the introductions to each of the mathematical chapters will give a measure of Erdös' talents and influence in a major area of mathematics. For the graph theorist, the mathematics describes the evolution of several important areas of the subject, through the work of an important figure and the work of his many, many collaborators. For the researcher in graph theory, this book will provide an accurate description of the status of certain topics and problems, while simultaneously providing direction about which new problems to pursue. Indeed, both the authors and the reviewer found it difficult not to feel the attraction of certain problems, and take a detour from reading to consider approaches to particular problems. If only as a matter of historical record, this book deserves a place in any comprehensive library of mathematics.

References

[1] J. Grossman and P. Ion, The Erdös Number Project, `http://www.acs.oakland.edu/~grossman/erdoshp.html`.

[2] P. Hoffman, The man who loves only numbers, Atlantic Monthly, v. 260, (5) 1987, pp. 60.

[3] P. Hoffman, The man who loved only numbers: the story of Paul Erdös and the search for mathematical truth. Hyperion, New York, 1998.

[4] B. Schechter, My brain is open: the mathematical journeys of Paul Erdös. Simon & Schuster, New York, 1998.

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An edited version of this review was published in *SIAM Review* 42, No. 1 (2000) as part of the Book Review section.