Math 232 | Name: | Dr. Beezer
Quiz #1 | | Fall 2003
Sections 1.1–1.5

Show *all* of your work and *explain* your answers fully. There is a total of 95 possible points.

1. Compute the following linear combination. (5 points)

$$3\begin{bmatrix}2\\-3\\4\\5\end{bmatrix} + (-6)\begin{bmatrix}1\\-1\\0\\3\end{bmatrix}$$

2. Find all solutions to each of the following systems of linear equations. Describe these sets of solutions using the vector form of the solutions where appropriate. You may use your calculator to row-reduce any matrices, being certain to write down the input matrix, the command used, and the output. (30 points)

(a)

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 5 \\ x_1 + 2x_2 - x_3 &= 4 \\ -x_1 + x_2 + 7x_3 &= -7 \end{aligned}$$

(b)

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 7x_4 &= -9 \\ x_1 + 2x_2 + 2x_3 + 5x_4 &= -5 \\ -x_1 - 2x_2 + x_3 + x_4 &= -7 \end{aligned}$$

3. A system of 5 equations has 4 variables. Describe the solution set under the following conditions. (15 points)
   (a) The reduced echelon form of the matrix representation of the system has 5 non-zero rows.

   (b) The system is homogeneous, and the reduced echelon form of the matrix representation of the system has 4 non-zero rows.

4. Perform the matrix-vector product computation below in two substantially different ways. (10 points)

$$\begin{bmatrix} 1 & -2 & 4 & -3 \\ 2 & 3 & -4 & 2 \\ -3 & 7 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 1 \\ 3 \end{bmatrix}$$

5. State a careful definition of **equivalent** systems of linear equations. (5 points)

6. Give conditions on b_1 and b_2 so that the system of equations below will be consistent. (15 points)

$$\begin{aligned} 3x_1 + 5x_2 &= b_1 \\ x_1 + 2x_2 &= b_2 \end{aligned}$$

7. Suppose that A is an $m \times n$ matrix and B is an $n \times p$ matrix. If B has a column of all zeros, prove that AB has a column of all zeros. (15 points)