Math 232 Name: Dr. Beezer
Quiz #2 Fall 2003
Sections 1.6, 1.7, 1.9

Show *all* of your work and *explain* your answers fully. There is a total of 90 possible points.

1. Determine, with complete explanations and justification, if the following sets of vectors are linearly independent or not. These will be graded as much on the explanation of your method as on the correctness of the results. (30 points)

(a) $\left\{ \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 7 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} -1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix} \right\}$

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 0 \\ -1 & 0 & 4 \end{bmatrix}$$

(a) Is A singular or nonsingular? (7 points)

(b) Does A have an inverse? Why or why not? If it does, compute it. (7 points)

(c) Consider the following system of equations. What do your answers to parts (a) and (b) tell you about solutions to this system? (10 points)

$$\begin{aligned} x_1 + x_2 - x_3 &= 2 \\ 2x_1 + 3x_2 \phantom{{}- x_3} &= 5 \\ -x_1 \phantom{{}+ x_2} + 4x_3 &= -7 \end{aligned}$$

(d) Find all solutions to the system in part (c), making as much use as possible of your work above. (6 points)

3. Suppose that A is a square matrix. Prove that AA^T is a symmetric matrix. (15 points)

4. Let A be the $n \times n$ matrix whose entries are ± 1 laid out in a "checkerboard" pattern. More precisely $[A]_{ij} = (-1)^{i+j}$. Discover, with proof, a formula for A^m. Hint: compute A^2 first. (15 points)