Math 232 | Name: | Dr. Beezer
Quiz #3 | | Fall 2003
Chapter 3

Show *all* of your work and *explain* your answers fully. There is a total of 90 possible points.

1. Consider the matrix A below. (30 points)

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 & 0 \\ -1 & 1 & -3 & -2 & 3 \\ 4 & 3 & 4 & 7 & -5 \end{bmatrix}$$

(a) Find a basis for the null space of A, $N(A)$.

(b) Find a basis for the range of A, $R(A)$.

(c) Find another basis for the range of A, $R(A)$, by using a *substantially* different method.

2. Let $S \subseteq \mathbb{R}^4$ be the set of vectors below. (12 points)

$$S = \{\mathbf{v_1},\, \mathbf{v_2},\, \mathbf{v_3}\} = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 1 \\ 3 \end{bmatrix} \right\}$$

(a) Is S an orthogonal set? An orthonormal set?

(b) Is S a basis of $\mathbb{R}^4$?

3. Suppose W is a subspace of $\mathbb{R}^5$ with dimension 3. Which of the following sets of vectors from W are bases of W? (18 points)

(a) $\left\{ \begin{bmatrix} -9 \\ 8 \\ 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 20 \\ -13 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} -9 \\ 8 \\ 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 18 \\ -13 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 9 \\ -5 \\ -1 \\ 2 \\ 0 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 7 \\ -5 \\ -3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 11 \\ 5 \\ 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -7 \\ 8 \\ 2 \\ 1 \\ -3 \end{bmatrix} \right\}$

4. Write a careful proof that $W = \{(x_1,\, x_2,\, x_3) \mid 3x_1 - 5x_2 + x_3 = 0\}$ is a subspace of $\mathbb{R}^3$. (15 points)

5. Suppose that $B = \{\mathbf{v_1},\, \mathbf{v_2},\, \ldots,\, \mathbf{v_m}\}$ is a basis of the subspace $V \subseteq \mathbb{R}^n$ (so in particular, $m \leq n$), and that $\mathbf{w} \in V$ is any vector from V. Prove that there is exactly one set of scalars $a_1,\, a_2, \ldots,\, a_m$ so that $\mathbf{w}$ can be written in the form $\mathbf{w} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 + \cdots + a_m\mathbf{v}_m$. (15 points)