Show *all* of your work and *explain* your answers fully. There is a total of 95 possible points.   You may use your calculator to row-reduce matrices, to find roots of polynomials, and to check your answers. Other computations (such as determinants and eigenvectors) should be done by hand.

1. Find the determinant of the matrix C below. (10 points)

$$C = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 5 & 2 & -1 \end{bmatrix}$$

2. Find the eigenvalues and eigenspaces of the matrix B below. (15 points)

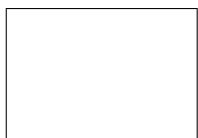$$B = \begin{bmatrix} 2 & -4 \\ 2 & -2 \end{bmatrix}$$

3. Consider the matrix A below. (30 points)

$$A = \begin{bmatrix} 7 & 0 & 4 \\ 8 & -1 & 4 \\ -8 & 0 & -5 \end{bmatrix}$$

(a) Compute the eigenvalues of A and their algebraic multiplicities.

(b) Compute the eigenspaces of A and the geometric multiplicities of the eigenvalues from part (a).

(c) Find an invertible matrix S that can be used to diagonalize A and specify the resulting diagonal matrix.

4. Suppose that P and Q are orthogonal matrices of the same size. Prove that PQ is also an orthogonal matrix. (10 points)

5. Suppose that A, B and C are matrices of the same size, such that A is similar to B and B is similar to C. Prove that A is similar to C. (15 points)

6. Suppose that A is a square matrix. Prove that the constant term of the characteristic polynomial of A is equal to $\det(A)$. (15 points)