Math 232 Name: Dr. Beezer

Quiz #5 Fall 2003

Sections 5.1–5.4

Show *all* of your work and *explain* your answers fully. There is a total of 90 possible points.

1. In P_3, the vector space of all polynomials of degree 3 or less, is the following set linearly independent or linearly dependent? (15 points)

   $\{x^3 + 3x^2 + 2x + 1,\ x^3 + x^2 - x + 1,\ 3x^3 + 5x^2 + 2x + 9,\ 2x^3 + 4x^2 - 1\}$

2. Does the set below span M_{22}, the vector space of all 2×2 matrices? (15 points)

   $$\left\{ \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}, \begin{bmatrix} -3 & 1 \\ -3 & 6 \end{bmatrix}, \begin{bmatrix} 4 & 3 \\ -2 & 4 \end{bmatrix} \right\}$$

3. In the vector space of all polynomials of degree 2 or less, P_2, consider the subspace $F = \{p(x) \in P_2 \mid p(5) = 0\}$. Find a basis for F and verify that it is linearly independent and spans F. (15 points)

4. Suppose that V is a vector space. Prove that $0\mathbf{x} = \mathbf{0}$, where $\mathbf{0}$ is the zero vector in V. Use only the 10 axioms for a vector space, and indicate clearly each time you use one. (15 points)

5. Let M_{88} be the set of all 8×8 matrices. Let S_8 be the set of all 8×8 skew-symmetric matrices. (Recall that a square matrix A is skew-symmetric if $A^T = -A$.) Prove that S_8 is a subspace of M_{88}. (15 points)

6. Suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_m\}$ is a linearly independent subset of the vector space V. Prove that $\{\mathbf{v_1}, \mathbf{v_1} + \mathbf{v_2}, \mathbf{v_1} + \mathbf{v_2} + \mathbf{v_3}, \ldots, \mathbf{v_1} + \mathbf{v_2} + \mathbf{v_3} + \ldots + \mathbf{v_m}\}$ is linearly independent. (15 points)