

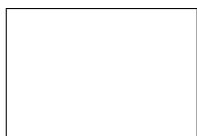
Show *all* of your work and *explain* your answers fully. There is a total of 90 possible points.

1. In  $P_3$ , the vector space of all polynomials of degree 3 or less, is the following set linearly independent or linearly dependent? (15 points)

$$\{x^3 + 3x^2 + 2x + 1, x^3 + x^2 - x + 1, 3x^3 + 5x^2 + 2x + 9, 2x^3 + 4x^2 - 1\}$$

2. Does the set below span  $M_{22}$ , the vector space of all  $2 \times 2$  matrices? (15 points)

$$\left\{ \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}, \begin{bmatrix} -3 & 1 \\ -3 & 6 \end{bmatrix}, \begin{bmatrix} 4 & 3 \\ -2 & 4 \end{bmatrix} \right\}$$



3. In the vector space of all polynomials of degree 2 or less,  $P_2$ , consider the subspace  $F = \{p(x) \in P_2 \mid p(5) = 0\}$ . Find a basis for  $F$  and verify that it is linearly independent and spans  $F$ . (15 points)

4. Suppose that  $V$  is a vector space. Prove that  $0\mathbf{x} = \mathbf{0}$ , where  $\mathbf{0}$  is the zero vector in  $V$ . Use only the 10 axioms for a vector space, and indicate clearly each time you use one. (15 points)



5. Let  $M_{88}$  be the set of all  $8 \times 8$  matrices. Let  $S_8$  be the set of all  $8 \times 8$  skew-symmetric matrices. (Recall that a square matrix  $A$  is skew-symmetric if  $A^T = -A$ .) Prove that  $S_8$  is a subspace of  $M_{88}$ . (15 points)

6. Suppose that  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \dots, \mathbf{v}_m\}$  is a linearly independent subset of the vector space  $V$ . Prove that  $\{\mathbf{v}_1, \mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3, \dots, \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 + \dots + \mathbf{v}_m\}$  is linearly independent. (15 points)

