Math 232 | Name: | Dr. Beezer
Quiz #6 | | Fall 2003
Sections 5.7–5.10

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.

1. Let $\mathbb{R}^3$ be the usual Euclidean vector space and let P_3 be the vector space of all polynomials with degree 3 or less. Consider the two bases $C = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix} \right\}$ and $D = \{1, x, x^2, x^3\}$.

   Define a linear transformation $T : \mathbb{R}^3 \to P_3$ by $T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = (2a+c)+(3a-4b+c)x-6cx^2+(a+b+c)x^3$.

   Form the matrix representation of T relative to C and D. (15 points)

2. Let P_1 be the vector space of polynomials with degree 1 or less, and let M_{12} be the vector space of 1×2 matrices. Define the function $T : P_1 \to M_{12}$ by $T(a+bx) = [3a-2b \quad a+4b]$. Prove that T is a linear transformation. (15 points)

3. Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined below. Is T one-to-one? Onto? (20 points)

$$T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} 3a + 2b - c \\ a - b + 4c \end{bmatrix}$$

4. Consider the linear transformation $S : P_2 \to P_2$ defined below. Find a basis of P_2 that will yield a diagonal representation of S, and specify that representation. (20 points)

$$S(a + bx + cx^2) = (3a - b - c) + (-12a + 5c)x + (4a - 2b - c)x^2$$

5. Suppose that U and V are finite-dimensional vector spaces with $\dim(U) < \dim(V)$ and that $T : U \to V$ is a linear transformation. Prove that T cannot be onto. (15 points)

6. Suppose that U and V are finite-dimensional vector spaces and $C = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \ldots, \mathbf{u}_n\}$ is a basis of U. Suppose that $T : U \to V$ and $S : U \to V$ are linear transformations such that $T(\mathbf{u}_i) = S(\mathbf{u}_i)$ for $1 \leq i \leq n$. Prove that $S = T$. (15 points)