Math 232 Name: Key Dr. Beezer
Quiz SLE Code: Spring 2005

Show *all* of your work and *explain* your answers fully. There is a total of 90 possible points.

1. Solve the linear system of equations below. (15 points)

$$\begin{aligned} x_1 + 2x_2 + 8x_3 - 7x_4 &= -2 \\ 3x_1 + 2x_2 + 12x_3 - 5x_4 &= 6 \\ -x_1 + x_2 + x_3 - 5x_4 &= -10 \end{aligned}$$

Solution: The augmented matrix of the system of equations is

$$\begin{bmatrix} 1 & 2 & 8 & -7 & -2 \\ 3 & 2 & 12 & -5 & 6 \\ -1 & 1 & 1 & -5 & -10 \end{bmatrix}$$

which row-reduces to

$$\begin{bmatrix} \boxed{1} & 0 & 2 & 1 & 0 \\ 0 & \boxed{1} & 3 & -4 & 0 \\ 0 & 0 & 0 & 0 & \boxed{1} \end{bmatrix}$$

With a leading one in the last column Theorem RCLS tells us the system of equations is inconsistent, so the solution set is the empty set, $\emptyset$.

2. Solve the linear system of equations below. (15 points)

$$\begin{aligned} 2x_1 + x_2 + 7x_3 - 2x_4 &= 4 \\ 3x_1 - 2x_2 + 11x_4 &= 13 \\ x_1 + x_2 + 5x_3 - 3x_4 &= 1 \end{aligned}$$

Solution: The augmented matrix of the system of equations is

$$\begin{bmatrix} 2 & 1 & 7 & -2 & 4 \\ 3 & -2 & 0 & 11 & 13 \\ 1 & 1 & 5 & -3 & 1 \end{bmatrix}$$

which row-reduces to

$$\begin{bmatrix} \boxed{1} & 0 & 2 & 1 & 3 \\ 0 & \boxed{1} & 3 & -4 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Then $D = 1, 2$ and $F = 3, 4, 5$, so the system is consistent ($5 \notin D$)and can be described by the two free variables x_3 and x_4. Rearranging the equations represented by the two nonzero rows to gain expressions for the dependent variables x_1 and x_2, yields the solution set,

$$S = \left\{ \left. \begin{bmatrix} 3 - 2x_3 - x_4 \\ -2 - 3x_3 + 4x_4 \\ x_3 \\ x_4 \end{bmatrix} \right| x_3,\, x_4 \in \mathbb{C} \right\}$$

3. Determine if the matrices below are singular or nonsingular. (15 points)

(a) $\begin{bmatrix} 2 & 3 & 1 & 4 \\ 1 & 1 & 1 & 0 \\ -1 & 2 & 3 & 5 \\ 1 & 2 & 1 & 3 \end{bmatrix}$

Solution: Row-reducing the matrix yields,

$$\begin{bmatrix} \boxed{1} & 0 & 0 & -2 \\ 0 & \boxed{1} & 0 & 3 \\ 0 & 0 & \boxed{1} & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since this is not the 4×4 identity matrix, Theorem NSRRI tells us the matrix is singular.

(b) $\begin{bmatrix} 9 & 3 & 2 & 4 \\ 5 & -6 & 1 & 3 \\ 4 & 1 & 3 & -5 \end{bmatrix}$

Solution: The matrix is not square, so neither term is applicable.

4. For each system of linear equations described below say as much as possible about the solution set. (15 points)

(a) Homogeneous, 7 equations in 7 variables.

Solution: Since the system is homogeneous, we know the zero vector is a solution (Theorem HSC). The solution set *could* be infinite, but we cannot determine this without the exact coefficient matrix.

(b) 12 equations in 35 variables.

Solution: The system could be inconsistent. If it is consistent, then Theorem CMVEI tells us the solution set will be infinite. So we can be certain that there is not a unique solution.

(c) 6 equations in 6 variables, singular coefficient matrix.

Solution: Theorem NSRRI tells us that the coefficient matrix will not row-reduce to the identity matrix. So if were to row-reduce the augmented matrix of this system of equations, we would not get a unique solution. So by Theorem PSSLS there remaining possibilities are no solutions, or infinitely many.

5. Each row operation is reversible. Using our shorthand notation for row operations, write down each of the three row operations in a general form. Then for each, write the row operation that "undoes" the original row operation. In other words, for each row operation provide the evidence that it is reversible by writing down the "reversing" row operation in a general form. (15 points)

Solution:

$R_i \leftrightarrow R_j$	$R_i \leftrightarrow R_j$
αR_i, $\alpha \neq 0$	$\frac{1}{\alpha} R_i$
$\alpha R_i + R_j$	$-\alpha R_i + R_j$

6. Prove that a linear system of equations is homogeneous if and only if the zero vector is a solution. (15 points)

   Solution: ($\Leftarrow$) Suppose we have a homogeneous system $\mathcal{LS}(A, \mathbf{0})$. The by substituting the scalar zero for each variable, we arrive at true statements for each equation. So the zero vector is a solution. This is the content of Theorem HSC.

   ($\Rightarrow$) Suppose now that we have a generic (i.e. not necessarily homogeneous) system of equations, $\mathcal{LS}(A, \mathbf{b})$ that has the zero vector as a solution. Upon substituting this solution into the system, we discover that each component of $\mathbf{b}$ must be also zero. So $\mathbf{b} = \mathbf{0}$.