Math 290
Quiz SLE

Name: Key

Dr. Beezer
Fall 2006

Show *all* of your work and *explain* your answers fully. There is a total of 90 possible points. If you use a calculator or Mathematica on a problem be sure to write down both the input and output.

1. Find all solutions to the system of equations below, and express your answer as a solution set. (15 points)

$$\begin{aligned} 2x_1 + x_2 + 3x_3 + 5x_4 &= -4 \\ x_1 + x_2 + x_3 + 4x_4 &= 1 \\ 3x_1 + 2x_2 + 4x_3 + 9x_4 &= -1 \end{aligned}$$

Augmented matrix:

$$\left[\begin{array}{cccc|c} 2 & 1 & 3 & 5 & -4 \\ 1 & 1 & 1 & 4 & 1 \\ 3 & 2 & 4 & 9 & -1 \end{array}\right]$$

Row reduces to

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 0 \\ 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

With a leading 1 in the final column we know that system is inconsistent (Thm RCLS)

Solution set: $\{\} = \emptyset$

2. Find all solutions to the system of equations below, and express your answer as a solution set. (15 points)

$$\begin{aligned} x_1 + -2x_2 + 2x_3 - 4x_4 &= 8 \\ -x_1 + 2x_2 + x_3 - 5x_4 &= -5 \\ x_1 - 2x_2 + 2x_4 &= 6 \end{aligned}$$

Augmented matrix

$$\left[\begin{array}{cccc|c} 1 & -2 & 2 & -4 & 8 \\ -1 & 2 & 1 & -5 & -5 \\ 1 & -2 & 0 & 2 & 6 \end{array}\right]$$

Row reduce to

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 2 & 6 \\ 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \longrightarrow x_1 = 6 + 2x_2 - 2x_4 \\ \longrightarrow x_3 = 1 + 3x_4 \\ \end{array}$$

$D = \{1, 3\}$

$F = \{2, 4, 5\}$

consistent

$\to x_2, x_4$ free

Solution set:

$$\{(6 + 2x_2 - 2x_4,\ x_2,\ 1 + 3x_4,\ x_4) \mid x_2, x_4 \in \mathbb{C}\}$$

3. Determine if the matrix below is singular or nonsingular. (15 points)

$$A = \begin{bmatrix} 1 & -1 & 3 & 2 \\ 1 & 3 & 2 & 0 \\ 1 & 3 & 3 & -1 \\ 2 & -2 & 7 & 3 \end{bmatrix}$$

row-reduce the matrix & apply Thm NSRRI

$$\xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & 4.25 \\ 0 & 1 & 0 & -0.75 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is not the 4x4 identity matrix, I_4 (Definition IM)

So A is not nonsingular

4. Convert the matrix below to reduced row-echelon form, doing all the computations by hand. In each step, indicate clearly which row operations you are performing. (15 points)

$$\begin{bmatrix} 1 & 2 & -4 & -4 \\ 1 & 1 & -3 & -3 \\ -2 & -1 & 6 & 8 \end{bmatrix} \xrightarrow[2R_1+R_3]{-R_1+R_2} \begin{bmatrix} 1 & 2 & -4 & -4 \\ 0 & -1 & 1 & 1 \\ 0 & 3 & -2 & 0 \end{bmatrix}$$

$$\xrightarrow{(-1)R_2} \begin{bmatrix} 1 & 2 & -4 & -4 \\ 0 & 1 & -1 & -1 \\ 0 & 3 & -2 & 0 \end{bmatrix} \xrightarrow[-3R_2+R_3]{-2R_2+R_1} \begin{bmatrix} 1 & 0 & -2 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\xrightarrow[2R_3+R_1]{R_3+R_2} \left[\begin{array}{ccc:c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

5. A three-digit number has two properties. The tens-digit and the ones-digit add up to 5. If the number is written with the digits in the reverse order, and then subtracted from the original number, the result is 792. Use a system of equations to find all of the three-digit numbers with these properties. (15 points)

Solution: Let a be the hundreds digit, b the tens digit, and c the ones digit. Then the first condition says that $b + c = 5$. The original number is $100a + 10b + c$, while the reversed number is $100c + 10b + a$. So the second condition is

$$792 = (100a + 10b + c) - (100c + 10b + a) = 99a - 99c$$

So we arrive at the system of equations

$$\begin{aligned} b + c &= 5 \\ 99a - 99c &= 792 \end{aligned}$$

Using row operations, or an augmented matrix, we arrive at the equivalent system

$$\begin{aligned} a - c &= 8 \\ b + c &= 5 \end{aligned}$$

We can vary c and obtain infinitely many solutions. However, c must be a digit, restricting us to ten values $(0 - 9)$. Furthermore, if $c > 2$, then the first equation causes $a > 9$, an impossibility. Setting $c = 0$, yields 850 as a solution, and setting $c = 1$ yields 941 as another solution.

6. Suppose that A is a singular matrix, and B is a matrix in reduced row-echelon form that is row-equivalent to A. Prove that the last row of B is a zero row. (15 points)

Solution: Let n denote the size of the square matrix A. By Theorem NSRRI the hypothesis that A is singular implies that B is not the identity matrix I_n. If B has n pivot columns, then it would have to be I_n, so B must have fewer than n pivot columns. But the number of nonzero rows in B (r) is equal to the number of pivot columns as well. So the n rows of B have fewer than n nonzero rows, and b must contain at least one zero row. By Definition RREF, this row must be at the bottom of B.