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Math 290
Proof SLE-1

Name:

Dr. R. Beezer
Fall 2012

- First due: **Friday, September 7**, *before* the start of class. Late submissions are not accepted.
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SLE-1 (Section HSE)

Suppose that the coefficient matrix of a homogeneous system of equations has a column of zeros. Prove that the system has infinitely many solutions.
Hint: What are the possibilities for the number of solutions to a linear system of equations?

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SLE-2 (Section NM)

Suppose that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2×2 matrix where $ad - bc \neq 0$. Prove that A is nonsingular.

Hints: One approach is to consider two cases, $a = 0$ and $a \neq 0$. No matter what approach you choose, make sure you are never dividing by a variable quantity that could be zero.

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V-1 (Section VO)

Prove Property AAC of Theorem VSPCV. That is:

If $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w} \in \mathbb{C}^m$, then $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$.

Write your own proof in the style of proofs of Property DSCA (Theorem VSPCV) and Property CC (Solution VO.T13).
Hints: Think carefully about the two types of equality you will likely use in a proof.

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V-2 (Section LI)

Prove that the set of standard unit vectors (Definition SUV) is linearly independent.

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M-1 (Section MM)

Suppose that A is an $m \times n$ matrix with a row where every entry is zero. Suppose that B is an $n \times p$ matrix. Prove that AB has a row where every entry is zero. Hints: Theorem EMP should be useful, and you want to be explicit about which row of A has the zeros and which row of AB has the zeros.
Hint: Which row is all zeros? Does it have a name?

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Proof M-2

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M-2 (Section MM)

Use Theorem EMP to prove part (2) of Theorem MMIM: If A is an $m \times n$ matrix and I_m is the identity matrix of size m, then $I_m A = A$.

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Math 290
Proof VS-1

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VS-1 (Section S)

Give an example of using Theorem TSS by proving that $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \middle| \, 5x_1 + 3x_2 - 8x_3 + 2x_4 = 0 \right\}$ is a subspace of $\mathbb{C}^4$.

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VS-2 (Section PD)

Carefully read Exercise PD.T60 and its solution (Solution PD.T60). Prove the "more general" result given in the solution, using the book's solution as a model.

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Math 290 Name: Dr. R. Beezer
Proof D&E-1 Fall 2012

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D&E-1 (Section DM)

Prove that the inverse of an elementary matrix is a single elementary matrix.
Hints: Seek inspiration from Exercise RREF.T10
This theorem does not say an elementary matrix is invertible — it says the inverse of an elementary matrix has the form of an elementary matrix — just one. How would you convince somebody of that?
There are three different types of elementary matrices, yes?

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D&E-2 (Section EE)

Suppose that A is a matrix that is equal to its inverse, $A = A^{-1}$. Prove that the only possible eigenvalues of A are $\lambda = 1$ and $\lambda = -1$. Give an example of matrix that is equal to its inverse and actually has both of these possible values as eigenvalues.
Hints: A theorem says that if λ is an eigenvalue of A, then λ^{-1} is an inverse of A^{-1}. The fact that $A = A^{-1}$ does not necessarily allow you to *immediately* conclude that $\lambda = \lambda^{-1}$.

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LT-1 (Section LT)

Prove that the function

$$T\colon\mathbb{C}^3\to\mathbb{C}^2,\quad T\left(\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right)=\begin{bmatrix}2x_1-x_2+5x_3\\3x_1+8x_3\end{bmatrix}$$

is a linear transformation.

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LT-2 (Section IVLT)

Suppose $T\colon U \to V$ is a surjective linear transformation and $\dim\left(U\right) = \dim\left(V\right)$. Prove that T is an invertible linear transformation.

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R-1 (Section MR)

Consider the two linear transformations,

$$T\colon M_{22} \to P_2, \qquad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (2a - b + 3c + d) + (2b - c + 2d)\,x + (4a - 2b + 3c + d)\,x^2$$

$$S\colon P_2 \to \mathbb{C}^2, \qquad S\left(p + qx + rx^2\right) = \begin{bmatrix} 2p + q - 3r \\ 5p + 2q - 4r \end{bmatrix}$$

and the bases of M_{22}, P_2 and $\mathbb{C}^2$ (respectively)

$$B = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ 1 & -4 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}, \begin{bmatrix} -2 & -4 \\ -5 & 4 \end{bmatrix} \right\}$$

$$C = \left\{1 + x,\ -2 - 3x + x^2,\ -2 - 2x + x^2\right\}$$

$$D = \left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$$

Verify the conclusion of Theorem MRCLT. In other words, build the three matrix representations of T, S and $S \circ T$ individually and check that they are related by the matrix product as in the theorem.

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R-2 (Section VR)

Let C be the crazy vector space from Section VS (Definition CVS). From Example DC, we know C has dimension 2. By Theorem CFDVS we can conclude that C must be isomorphic to $\mathbb{C}^2$. Construct a function $T\colon \mathbb{C}^2 \rightarrow C$ that is a candidate for an isomophism between these two vector spaces by giving an *explicit* formula for T. Then give a convincing argument that T is indeed an isomorphism.

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