Math 290   Name: Dr. R. Beezer
Proof SLE-1   Spring 2012

Pass   Retries: No Pass

- First due date: **Friday, February 3**, before the start of class as a PDF attachment to an email. Late submissions will not be accepted.

- Use `http://tex.mendelu.cz/en/` for the quickest way to make a PDF.
  Make your own backups for retries, do not trust the storage at this site.

- As a graded assignment, **this is your own work**.
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- Present your final version of your proof in a file all by itself. You do not have to restate the problem, but do include the title of the problem, i.e. SLE-1.

- Do not include any scratch work. Just present a final version.

- Write with complete sentences.

- Retry: only use theorems and definitions from the relevant chapter, or earlier.

- Retry: submit a complete, new version.

SLE-1 (Section HSE)

Suppose that the coefficient matrix of a homogeneous system of equations has a column of zeros. Prove that the system has infinitely many solutions.

Math 290   Name: Dr. R. Beezer
Proof SLE-2   Spring 2012

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SLE-2 (Section NM)

Suppose that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2×2 matrix where $ad - bc \neq 0$. Prove that A is nonsingular.

Hints: One approach is to consider two cases, $a = 0$ and $a \neq 0$. No matter what approach you choose, think carefully about the possibility of dividing by zero throughout your proof.

Math 290   Name: Dr. R. Beezer
Proof V-1   Spring 2012

PASS   RETRIES: NO PASS

- First due date: **Monday, February 20**, before the start of class as a PDF attachment to an email. Late submissions will not be accepted.

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V-1 (Section VO)

Prove Property AAC of Theorem VSPCV. That is:

If $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w} \in \mathbb{C}^m$, then $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$.

Write your own proof in the style of proofs of Property DSCA (Theorem VSPCV) and Property CC (Solution VO.T13).

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V-2 (Section LI)
Prove that the set of standard unit vectors (Definition SUV) is linearly independent.

Pass   Retries: No Pass

- First due date: **Tuesday, March 6**, before the start of class as a PDF attachment to an email. Late submissions will not be accepted.

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- Present your final version of your proof in a file all by itself. You do not have to restate the problem, but do include the title of the problem, i.e. M-1.

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M-1 (Section MM)

Suppose that A is an $m \times n$ matrix with a row where every entry is zero. Suppose that B is an $n \times p$ matrix. Prove that AB has a row where every entry is zero. Hints: Theorem EMP should be useful, and you want to be explicit about which row of A has the zeros and which row of AB has the zeros.

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M-2 (Section MM)

Use Theorem EMP to prove part (2) of Theorem MMIM: If A is an $m \times n$ matrix and I_m is the identity matrix of size m, then $I_m A = A$.

- First due date: **Thursday, March 29**, before the start of class as a PDF attachment to an email. Late submissions will not be accepted.

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VS-1 (Section S)

Give an example of using Theorem TSS by proving that $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| 5x_1 + 3x_2 - 8x_3 = 0 \right\}$ is a subspace of $\mathbb{C}^3$.

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VS-2 (Section PD)

Carefully read Exercise PD.T60 and its solution (Solution PD.T60). Prove the "more general" result given in the solution, using the book's solution as a model.

Pass　　　　　　　　Retries:　　　　　　　　No Pass

- First due date: **Tuesday, April 10**, before the start of class as a PDF attachment to an email. Late submissions will not be accepted.

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- Write with complete sentences.

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D&E-1 (Section DM)

Prove that the inverse of an elementary matrix is a single elementary matrix. Hint: seek inspiration from Exercise RREF.T10

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- Present your final version of your proof in a file all by itself. You do not have to restate the problem, but do include the title of the problem, i.e. D&E-2.

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- Write with complete sentences.

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- Retry: submit a complete, new version.

D&E-2 (Section EE)

Suppose that A is a matrix that is equal to its inverse, $A = A^{-1}$. Prove that the only possible eigenvalues of A are $\lambda = 1$ and $\lambda = -1$. Give an example of matrix that is equal to its inverse and actually has both of these possible values as eigenvalues.

PASS   RETRIES: NO PASS

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- Write with complete sentences.

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LT-1 (Section LT)
Prove that the function

$$T\colon \mathbb{C}^3 \to \mathbb{C}^2, \quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 + 5x_3 \\ 3x_1 + 8x_3 \end{bmatrix}$$

is a linear transformation.

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LT-2 (Section IVLT)

Suppose $T : U \to V$ is a surjective linear transformation and $\dim(U) = \dim(V)$. Prove that T is an invertible linear transformation.

- First due date: **Monday, April 30**, before the start of class as a PDF attachment to an email. Late submissions will not be accepted.

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R-1 (Section MR)

Consider the two linear transformations,

$$T\colon M_{22} \to P_2, \qquad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (2a - b + 3c + d) + (2b - c + 2d)\,x + (4a - 2b + 3c + d)\,x^2$$

$$S\colon P_2 \to \mathbb{C}^2, \qquad S\left(p + qx + rx^2\right) = \begin{bmatrix} 2p + q - 3r \\ 5p + 2q - 4r \end{bmatrix}$$

and the bases of M_{22}, P_2 and $\mathbb{C}^2$ (respectively)

$$B = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ 1 & -4 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}, \begin{bmatrix} -2 & -4 \\ -5 & 4 \end{bmatrix} \right\}$$

$$C = \left\{ 1 + x,\ -2 - 3x + x^2,\ -2 - 2x + x^2 \right\}$$

$$D = \left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$$

Verify the conclusion of Theorem MRCLT. In other words, build the three matrix representations of T, S and $S \circ T$ individually and check that they are related by the matrix product as in the theorem.

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R-2 (Section VR)

Let C be the crazy vector space from Section VS (Definition CVS). From Example DC, we know C has dimension 2. By Theorem CFDVS we can conclude that C must be isomorphic to $\mathbb{C}^2$. Construct a function $T\colon \mathbb{C}^2 \to C$ that is a candidate for an isomophism between these two vector spaces by giving an *explicit* formula for T. Then give a convincing argument that T is indeed an isomorphism.