Math 290 Name: Key Dr. Beezer
Exam 1 Spring 2012
Chapter SLE

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Use Sage only to row-reduce matrices, and not at all for the one question where Sage is banned.

1. Find the solution set for the following system of linear equations. (15 points)

$$\begin{aligned} 2x_1 + 5x_2 - 3x_3 + 4x_4 &= 10 \\ 7x_1 + x_2 + 7x_3 + 2x_4 &= -10 \\ 6x_1 - x_2 - x_3 - 2x_4 &= 18 \\ -x_1 + 3x_2 + 8x_3 + 5x_4 &= -33 \end{aligned}$$

Form the augmented matrix of the system and row-reduce to:

$$\xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

So an equivalent system is

$x_1 = 2$
$x_2 = 1$
$x_3 = -3$
$x_4 = -2$

The solution <u>set</u> is $\left\{ \begin{bmatrix} 2 \\ 1 \\ -3 \\ -2 \end{bmatrix} \right\}$

2. Find the solution set for the following system of linear equations. (15 points)

$$\begin{aligned} 3x_1 + x_2 + 4x_3 + 3x_4 - x_5 &= 0 \\ -x_2 + 3x_3 + 8x_4 - 3x_5 &= 2 \\ -x_1 - x_2 + x_3 + 5x_4 - 2x_5 &= 1 \\ 4x_1 + x_2 + 5x_3 + 4x_4 - x_5 &= 2 \end{aligned}$$

Form the augmented matrix of the system and row-reduce to:

$$\xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 3 \\ 0 & 1 & 0 & -2 & 0 & 5 \\ 0 & 0 & 1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So an equivalent system is:

$x_1 = 3 + x_4 - x_5$
$x_2 = 5 + 2x_4$
$x_3 = -1 - 2x_4 + x_5$

The solution set is

$$\left\{ \begin{bmatrix} 3 + x_4 - x_5 \\ 5 + 2x_4 \\ -1 - 2x_4 + x_5 \\ x_4 \\ x_5 \end{bmatrix} \middle| \, x_4, x_5 \in \mathbb{C} \right\}$$

3. Without using Sage, find a matrix B that is reduced row-echelon form and is row-equivalent to A. (15 points)

$$A = \begin{bmatrix} 1 & -2 & -5 & -3 \\ 0 & 1 & 0 & -3 \\ -1 & 0 & 4 & 7 \end{bmatrix} \xrightarrow{R_1+R_3} \begin{bmatrix} 1 & -2 & -5 & -3 \\ 0 & 1 & 0 & -3 \\ 0 & -2 & -1 & 4 \end{bmatrix}$$

$$\xrightarrow[2R_2+R_3]{2R_2+R_1} \begin{bmatrix} 1 & 0 & -5 & -9 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & -1 & -2 \end{bmatrix} \xrightarrow{-1R_3} \begin{bmatrix} 1 & 0 & -5 & -9 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\xrightarrow{5R_3+R_1} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

4. Find the null space of the matrix F, $\mathcal{N}(F)$. (15 points)

$$F = \begin{bmatrix} 0 & -1 & 4 & 7 \\ -3 & 4 & -7 & -4 \\ -4 & 4 & -3 & 6 \\ -1 & 0 & 3 & 8 \end{bmatrix}$$

Form the augmental matrix $[F \mid \underset{\sim}{0}]$ of $\mathcal{LS}(A, \underset{\sim}{0})$ and row-reduce to

$$\xrightarrow{RREF} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

An equivalent system:

$x_1 = 2x_4$
$x_2 = -x_4$
$x_3 = -2x_4$

$$N(F) = \text{Solution set to } \mathcal{LS}(F, \underset{\sim}{0}) = \left\{ \begin{bmatrix} 2x_4 \\ -x_4 \\ -2x_4 \\ x_4 \end{bmatrix} \;\middle|\; x_4 \in \mathbb{C} \right\}$$

5. Determine if the matrix C is singular or nonsingular. (15 points)

$$C = \begin{bmatrix} -2 & -1 & -4 & 5 \\ 3 & 2 & 8 & -6 \\ -3 & -2 & -7 & 8 \\ 3 & 1 & 6 & -6 \end{bmatrix}$$

An easy test is to see if the matrix row reduces to the identity matrix (Theorem NMRRI)

$$C \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since this is the 4x4 identity matrix, we conclude that C is non singular.

6. Suppose that two systems of linear equations are equivalent. Prove that if one system is homogeneous, then the other must also be homogeneous. (15 points)

The first system is homogeneous, hence the zero vector is a solution. (This is the content of Theorem HSC.)

The systems are equivalent, so the zero vector is a solution to the second system.

If we substitute zero for each variable, we discover that the vector of constants is all zeros, ie the system is homogeneous.