Show all of your work and explain your answers fully. There is a total of 100 possible points.

Use Sage only to row-reduce matrices or to compute extended echelon form, except where indicated in a problem statement. Include the results of these computations in your answers and describe the input used.

1. Other than actually asking Sage to compute the inverse of the matrix A below, what Sage command could you use to see that A has an inverse? Then compute the inverse of A only using just the reduced row-echelon form command, .rref(), on an appropriate matrix. (You can create the "appropriate" matrix with any Sage commands you like.) (15 points)

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$$A = \begin{bmatrix} -2 & -3 & -1 & 0 \\ 3 & 4 & 2 & -2 \\ -3 & -4 & -1 & 0 \\ -4 & -6 & -4 & 5 \end{bmatrix}$$

A. inverse() gives

$$A' = \begin{bmatrix} 0 & 5 & 2 & 2 \\ 1 & -5 & 3 & -2 \\ 4 & 5 & 5 & 2 \end{bmatrix}$$

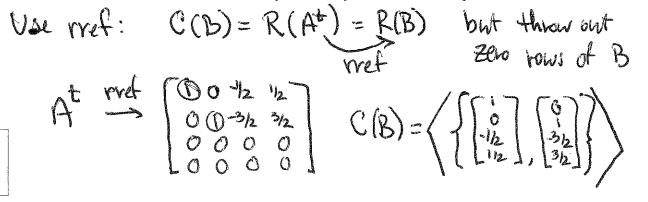
2. Determine a linearly independent spanning set for the column space of B in two different ways, meeting the requirements given. (20 points)

$$B = \begin{bmatrix} 0 & -1 & -1 & 2 & -1 \\ -2 & 7 & 3 & -16 & 13 \\ 3 & -10 & -4 & 23 & -19 \\ -3 & 10 & 4 & -23 & 19 \end{bmatrix} \qquad \begin{array}{c} \text{RVEF} \\ \text{O} & \text{O} & 2 & 1 & -3 \\ \text{O} & \text{O} & 1 & -2 & 1 \\ \text{O} & \text{O} & \text{O} & \text{O} & \text{O} \\ \text{O} & \text{O} & \text{O} & \text{O} & \text{O} \\ \end{array}$$

(a) The set contains only vectors that are columns of B.

$$D = \frac{31,21}{\text{prot columns}}$$
 By Theorem BCS:
 $C(B) = \left\{ \left[\frac{-3}{3} \left[\frac{-10}{10} \right] \right\} \right\}$

(b) The set should be obtained in the most computationally efficient manner possible.



- 3. Consider the matrix B from the previous question. (20 points)
 - (a) Use the matrix L from the extended echelon form to compute a linearly independent set whose span equals the column space of B.

$$\begin{bmatrix}
B | T_{i} \\
C(B) = N(L) = \left(\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} = \left(\left\{ \begin{bmatrix} -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right\} \right)$$

(b) Use the matrix L from the extended echelon form to compute a linearly independent set whose span equals the left null space of B.

L(B) = R(L) =
$$\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ 1 \end{bmatrix} \right\}$$

Non zero rows of L
Written as columns

4. Suppose that A and B are both $m \times n$ matrices. Prove that $(A + B)^t = A^t + B^t$. (15 points)

- 5. When computing the extended echelon form of an $m \times n$ matrix A we compute $[A|I_m] \xrightarrow{\text{RREF}} [B|J]$. Prove that J is nonsingular. (15 points)
 - 1 See the first paragraph of the proof of themen PEEF.
 - E) Is I now-equardent to Im? What is the sequence of now operations? We need to know that the now operations which convert Im to I can be reverted to create I to Im. See Exercise RFEF. TIO.
- 6. Suppose $\alpha \in \mathbb{C}$ is a scalar, A is an $m \times n$ matrix and B is an $n \times p$ matrix. Prove that $(\alpha A)B = A(\alpha B)$. (15 points)