Math 290 Name: Dr. Beezer
Exam 2 Spring 2012
Chapter V

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Use Sage only to row-reduce matrices and include these computations in your answers.

1. Determine if the vector $\mathbf{y}$ is in the span of the set S, $\langle S\rangle$. (15 points)

$$\mathbf{y} = \begin{bmatrix}2\\-11\\1\\7\end{bmatrix} \qquad S = \{\mathbf{v}_1,\,\mathbf{v}_2,\,\mathbf{v}_3,\,\mathbf{v}_4,\,\mathbf{v}_5,\,\mathbf{v}_6\} = \left\{\begin{bmatrix}1\\0\\-2\\0\end{bmatrix},\,\begin{bmatrix}-1\\1\\1\\-1\end{bmatrix},\,\begin{bmatrix}1\\-5\\3\\5\end{bmatrix},\,\begin{bmatrix}-1\\-4\\3\\2\end{bmatrix},\,\begin{bmatrix}2\\4\\-5\\-2\end{bmatrix},\,\begin{bmatrix}4\\6\\5\\-4\end{bmatrix}\right\}$$

2. Determine if the sets of vectors below are linearly independent or not. Be sure to provide sufficient justification. (20 points)

(a) $$\left\{\begin{bmatrix}1\\0\\1\\-1\\1\end{bmatrix},\,\begin{bmatrix}2\\1\\0\\-1\\-3\end{bmatrix},\,\begin{bmatrix}7\\2\\4\\0\\1\end{bmatrix}\right\}$$

(b) $$\left\{\begin{bmatrix}1\\-2\\0\\0\\1\end{bmatrix},\,\begin{bmatrix}-6\\1\\-3\\1\\2\end{bmatrix},\,\begin{bmatrix}-8\\5\\-3\\1\\0\end{bmatrix}\right\}$$

3. The set S below is the same as in Question 1. Find a linearly independent set T so that $\langle T \rangle = \langle S \rangle$. (10 points)

$$S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5, \mathbf{v}_6\} = \left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ -5 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 5 \\ -4 \end{bmatrix} \right\}$$

4. The vector $\mathbf{y}$ below is the same as in Question 1. Find a linear combination of the vectors in the set set T (that you found in the previous question) that equals $\mathbf{y}$. Comment thoughtfully on the relationship between the results in Question 1, the previous question, and this question. (10 points)

$$\mathbf{y} = \begin{bmatrix} 2 \\ -11 \\ 1 \\ 7 \end{bmatrix}$$

5. Find a linearly independent set R whose span is the null space of the matrix A below. In other words, R will be linearly independent and $\langle R \rangle = \mathcal{N}(A)$. (10 points)

$$\begin{bmatrix} 1 & -3 & 7 & -4 & 4 \\ 1 & -2 & 3 & -2 & 1 \\ 0 & 2 & -8 & 5 & -8 \\ 0 & -1 & 4 & 2 & -5 \end{bmatrix}$$

6. Given two vectors $\mathbf{u}$, $\mathbf{v} \in \mathbb{C}^m$ define a new operation, called *subtraction*, by $[\mathbf{u} - \mathbf{v}]_i = [\mathbf{u}]_i - [\mathbf{v}]_i$, $1 \leq i \leq m$. Prove that subtraction is not really anything new (because we can accomplish subtraction with operations we already have) by showing that $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-1)\mathbf{v}$. (10 points)

7. Referring to the result about subtraction from the previous question, prove that for $\alpha \in \mathbb{C}$ and $\mathbf{u}$, $\mathbf{v} \in \mathbb{C}^m$, $\alpha\,(\mathbf{u} - \mathbf{v}) = \alpha\mathbf{u} - \alpha\mathbf{v}$. (10 points)

8. Suppose that $\mathbf{v}_1$, $\mathbf{v}_2 \in \mathbb{C}^m$. Prove that $\langle\{\mathbf{v}_1,\, \mathbf{v}_2\}\rangle = \langle\{\mathbf{v}_1 + \mathbf{v}_2,\, \mathbf{v}_1 - \mathbf{v}_2\}\rangle$. (10 points)