Math 290 Name: Dr. Beezer
Exam 4 Spring 2012
Chapter VS

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Use Sage only to row-reduce matrices or to solve systems of equations, and be sure to detail your input and output.

1. Determine if the set R below is a linearly independent set in P_2, the vector space of polynomials of degree at most two. (15 points)

   $R = \left\{x^2 + 3x - 5,\ 3x^2 - 8x + 2\right\}$

2. Find a spanning set for the subspace Y of M_{22}. (15 points)

   $Y = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{22} \,\middle|\, a = 4c,\ b = -2d \right\}$

3. For the matrix A below, compute the rank, nullity and the dimension of the column space. (5 points)

$$A = \begin{bmatrix} 1 & -1 & 6 & 0 & 1 & -1 & -3 \\ 0 & 1 & -3 & 2 & -5 & 5 & -2 \\ -2 & 0 & -6 & -3 & 5 & -5 & 8 \\ 0 & 1 & -3 & 2 & -5 & 5 & -2 \\ 0 & -2 & 6 & -3 & 7 & -7 & 2 \end{bmatrix}$$

4. Consider the subspace W of P_3 (the vector space of all polynomials of degree at most 3), and the four elements $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, $\mathbf{y}$ of W. You may assume the following: W is a subspace, all four elements below are in W (except for when you do part (a)), and $B = \{\mathbf{u}, \mathbf{y}\}$ is a basis of W. (35 points)

$$W = \left\{ a + bx + cx^2 + dx^3 \middle| \, 7a + 5b - 4c + 3d = 0, 4a + 3b - 2c + 2d = 0 \right\}$$

$$\mathbf{u} = 3 + 2x + 4x^2 - 5x^3 \quad \mathbf{v} = -1 + 4x + x^2 - 3x^3 \quad \mathbf{w} = -5 + 2x - 4x^2 + 3x^3 \quad \mathbf{y} = 2 + 6x + 5x^2 - 8x^3$$

(a) Verify that one of $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, $\mathbf{y}$ is an element of W (your choice, just one).

(b) Does the set $T = \{\mathbf{w}\}$ span W? Why or why not?

(c) Is the set $R = \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ a linearly independent subset of W? Why or why not?

(d) Does the set $T = \{\mathbf{u}, \mathbf{w}\}$ span W? Why or why not?

5. Suppose that V is a vector space. Prove that the zero vector of V is unique. (15 points)

6. Suppose that A is an $m \times m$ nonsingular matrix and that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_p\}$ is a linearly independent subset of $\mathbb{C}^m$. Prove that $T = \{A\mathbf{v}_1, A\mathbf{v}_2, \ldots, A\mathbf{v}_p\}$ is a linearly independent subset of $\mathbb{C}^m$. (15 points)