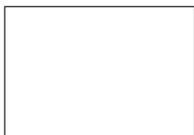


3. Diagonalize the matrix C below by giving the matrix for the similarity transformation and the resulting diagonal matrix. Use **only** the Sage `.eigenspaces_right()` method and show clearly how you are using the output. (20 points)

$$C = \begin{bmatrix} -16 & 21 & 42 & 42 \\ -14 & 19 & 28 & 28 \\ 7 & -7 & -9 & -14 \\ -7 & 7 & 14 & 19 \end{bmatrix}$$

4. Suppose that \mathbf{x} and \mathbf{y} are eigenvectors for the same eigenvalue λ of the matrix A . Prove directly that $\mathbf{x} + \mathbf{y}$ is also an eigenvector for the eigenvalue λ of the matrix A . Here, “directly” means you are proving a closure property, and to not use the fact that eigenspaces are vector spaces. (15 points)



5. Suppose that A has $\lambda = 4$ as an eigenvalue. Define the polynomial $p(x) = 2x^3 - 6x - 31$. Find, with proof, an eigenvalue of $p(A)$. (15 points)

6. Use Sage to create some small, but nontrivial, square matrices over the rationals. For each matrix, compute the characteristic polynomial, and then evaluate the polynomial with the matrix. Document your experiments by giving the relevant commands employed and the output created. What do you observe? (15 points)

