Math 290 Name: Dr. Beezer
Exam 7 Spring 2012
Chapter R

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use Sage to row-reduce matrices, solve systems of equations, compute determinants and compute eigenstuff. Linear transformation routines may not be used as justification.

1. Compute a matrix representation of the linear transformation T with domain P_1, the vector space of polynomials with degree at most 1, and codomain M_{13}, the vector space of 1×3 matrices. Then illustrate the Fundamental Theorem of Matrix Representation (FTMR) by using the representation to compute $T(3-6x)$. (No credit will be given for using other methods to compute this output of the linear transformation.) (15 points)

   $T\colon P_1 \to M_{13},\quad T(a+bx) = \begin{bmatrix} 2a+b & a-4b & 5a+6b \end{bmatrix}$

2. For the linear transformation below, find a basis of the vector space P_2 so that the matrix representation of S relative to the basis is a diagonal matrix. Give the ensuing representation as well. (20 points)

   $S\colon P_2 \to P_2,\quad S\left(a+b\,x+c\,x^2\right) = (-12a-4b+14c) + (9a+6b-9c)\,x + (-9a-4b+11c)\,x^2$

3. Consider the linear transformation R with domain M_{12}, the vector space of 1×2 matrices, and codomain P_1, the vector space of polynomials with degree at most 1. (35 points)

$R\colon M_{12} \to P_1, \quad R\left(\begin{bmatrix} a & b \end{bmatrix}\right) = (3a + 7b) + (2a + 5b)\,x$

(a) Build a matrix representation of R relative to the bases $B = \left\{\begin{bmatrix} 3 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 3 \end{bmatrix}\right\}$ and $C = \{6 + 5x,\ 1 + x\}$.

(b) R is invertible (you may assume this). Compute the matrix representation of R^{-1} relative the bases C and B given in part (a).

(c) Consider two new bases, $X = \left\{\begin{bmatrix} 7 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 1 \end{bmatrix}\right\}$ and $Y = \{1 + 2x,\ -1 - x\}$. Form the change-of-basis matrix, $C_{B,X}$, from basis B to basis X.

(d) Compute the matrix representation of R relative to the bases X and Y (in part (c)) by using the representation from part (a) and change-of-basis matrices. No credit will be given for building this representation via the definition of a matrix representation.

4. We saw the following linear transformation on the previous exam. Suppose U is a vector space and $\rho \in \mathbb{C}$ is a scalar. Define the linear transformation $T_\rho \colon U \to U$ by $T_\rho(\mathbf{u}) = \rho\mathbf{u}$. Describe, with justification, a matrix representation of T. (15 points)

5. Suppose that U is a vector space, B is a basis of U, $T \colon U \to U$ is a linear transformation, and $\mathbf{u} \in U$ is an eigenvector of T. Prove that the vector representation $\rho_B(\mathbf{u})$ is an eigenvector for the matrix representation $M^T_{B,B}$. (15 points)