Math 290 Name: Dr. Beezer
Exam 1 Fall 2013
Chapter SLE

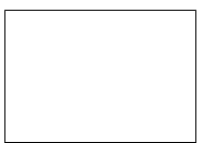
Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use Sage to row-reduce matrices, except in the question that asks you to row-reduce without Sage. No other use of Sage may be used as justification for your answers.

1. Solve the following system of linear equations and express the solutions as a set of column vectors. (15 points)

$$\begin{aligned} 4x_1 - 3x_2 + 8x_3 &= -3 \\ -x_1 + x_2 - 2x_3 &= 1 \\ 4x_1 - 4x_2 + 8x_3 &= -4 \\ 2x_1 + x_2 + 4x_3 &= 2 \end{aligned}$$

2. Solve the following system of linear equations and express the solutions as a set of column vectors. (20 points)

$$\begin{aligned} x_1 + x_2 + 3x_4 + 2x_5 + 4x_6 &= 16 \\ x_2 - 2x_3 + 3x_4 + x_5 + 3x_6 &= 4 \\ x_1 + 2x_3 + x_5 + x_6 &= 12 \\ -x_1 + x_2 - 4x_3 + 3x_4 + 2x_6 &= -8 \end{aligned}$$



3. Determine if the matrix below is nonsingular or singular. Explain your reasoning carefully and thoroughly. (15 points)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 2 & -3 \\ -1 & 1 & 1 & 0 & 0 & 2 & 3 & -3 \\ 1 & 0 & -5 & 1 & -2 & 3 & -3 & 5 \\ -1 & 0 & 5 & 0 & 1 & 0 & 2 & -4 \\ -1 & 0 & 5 & 1 & 0 & 3 & 1 & -3 \\ 1 & 0 & -5 & -1 & 3 & -6 & 5 & -6 \\ 1 & 0 & -5 & -1 & 0 & -3 & -1 & 3 \\ -2 & 1 & 6 & -1 & 0 & 1 & 2 & -2 \end{bmatrix}$$

4. Without using Sage, find a matrix B in reduced row-echelon form which is row-equivalent to A. It is especially important to show all of your work, so it is clear you have not used Sage. (20 points)

$$A = \begin{bmatrix} 1 & 1 & -1 & -6 & 2 \\ -1 & 0 & -2 & 4 & -2 \\ -2 & 0 & -4 & 9 & -3 \end{bmatrix}$$

5. Find all of the values of α for which the following system has a unique solution. (15 points)

$$\begin{aligned} x_1 - 3\,x_2 - \ x_3 &= 3 \\ -2\,x_1 + x_2 - 3\,x_3 &= -1 \\ x_1 + 2\,x_2 + \alpha\,x_3 &= -1 \end{aligned}$$

6. Suppose that: (1) A is an $n \times n$ square matrix, (2) $\mathbf{b}$ is a vector with n entries and (3) $\mathcal{LS}(A, \mathbf{b})$ has a unique solution. Prove that A is nonsingular. (15 points)