Math 290 Name: Dr. Beezer
Exam 2 Fall 2013
Chapter V

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use Sage to row-reduce matrices so long as you explain your input and show your output in your work.
No other use of Sage may be used as justification for your answers.

1. Determine if the vector $\mathbf{y}$ is in the span of S, along with a complete explanation. (15 points)

$$\mathbf{y} = \begin{bmatrix} -3 \\ -8 \\ 0 \\ 6 \end{bmatrix} \qquad S = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ -7 \\ 1 \end{bmatrix} \right\}$$

2. Is the set of vectors R linearly independent or linearly dependent? Explain why. (10 points)

$$R = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ -5 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} -8 \\ -4 \\ -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 4 \\ 3 \\ -3 \end{bmatrix} \right\}$$

3. Express solutions to the following system in "vector form" as described in Theorem VFSLS ("Vector Form of Solutions to Linear Systems"). (15 points)

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 - 5\,x_5 + 8\,x_6 &= -15 \\ x_2 + 3\,x_4 - 6\,x_5 + 2\,x_6 &= -8 \\ -x_1 + 5\,x_4 + 4\,x_5 - 3\,x_6 &= 38 \\ x_2 + x_3 + 6\,x_4 - x_5 + 6\,x_6 &= 22 \end{aligned}$$

4. For the set of vectors R below, find a set of vectors T meets the following two requirements and explain why your set T meets these requirements. (15 points)

   (a) $T = \langle R \rangle$

   (b) T is linearly independent

$$R = \left\{ \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\-2\\-2\end{bmatrix}, \begin{bmatrix}3\\4\\-8\\-8\end{bmatrix}, \begin{bmatrix}1\\-3\\7\\2\end{bmatrix}, \begin{bmatrix}-2\\-3\\6\\6\end{bmatrix}, \begin{bmatrix}2\\2\\-4\\-4\end{bmatrix} \right\}$$

5. Find a linear combination of vectors from S which equals $\mathbf{w}$, *using as few vectors as possible.* Include an explanation of why you are *sure* you have used the fewest possible. (15 points)

$$\mathbf{w} = \begin{bmatrix}5\\4\\-4\\-3\end{bmatrix} \qquad S = \left\{ \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\-1\\-1\end{bmatrix}, \begin{bmatrix}1\\4\\3\\3\end{bmatrix}, \begin{bmatrix}0\\-3\\-3\\-3\end{bmatrix}, \begin{bmatrix}0\\2\\-1\\0\end{bmatrix} \right\}$$

6. Suppose that $\alpha \in \mathbb{C}$ and $\mathbf{u} \in \mathbb{C}^m$. Give a careful proof that $\overline{(\alpha \mathbf{u})} = \overline{\alpha}\,\overline{\mathbf{u}}$. (Notice that this is Theorem CRSM so you are being asked to do more than just quote the statement of the theorem.) (15 points)

7. Suppose that $\mathbf{x},\, \mathbf{y} \in \mathbb{C}^m$, $S = \langle\{\mathbf{x},\, \mathbf{y}\}\rangle$ and $T = \langle\{7\mathbf{x} + 3\mathbf{y},\, 2\mathbf{x} + \mathbf{y}\}\rangle$. Prove that $S = T$. (15 points)