

Writing Exercises

Math 290A Spring 2014

Dr. Beezer

General Reminders and Hints:

1. Use math-mode for every symbol, even if it is just a single letter, so use x for the variable x .
2. Vectors must be typeset differently: \vec{v} or \mathbf{v} .
3. “A 3×4 matrix” can be accomplished with 3×4 .
4. `\usepackage{amsmath}` and `\usepackage{amssymb}` are necessary in the preamble to enable many extra mathematical commands.
5. For a matrix try

```

$A=\begin{bmatrix}
a & b & c & d \\
e & f & g & h \\
i & j & k & \ell
\end{bmatrix}$

```

to get $A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & \ell \end{bmatrix}$

6. For a proof with reasons try

```

\begin{align*}
x^2 - y^2 &= x^2 - xy + xy - y^2 && \text{\text{Adding zero}} \\
&= x(x-y) + y(x-y) && \text{\text{Distributivity, twice}} \\
&= (x+y)(x-y) && \text{\text{Distribute } x-y}
\end{align*}

```

to get

$x^2 - y^2 = x^2 - xy + xy - y^2$	Adding zero
$= x(x - y) + y(x - y)$	Distributivity, twice
$= (x + y)(x - y)$	Distribute $x - y$

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SLE-1 (Section HSE)

Suppose that the coefficient matrix of a homogeneous system of equations has a column of zeros. Prove that the system has infinitely many solutions.

Hint: What are the possibilities for the number of solutions to a linear system of equations? Can you definitively rule out any of these?

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SLE-2 (Section NM)

Suppose that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2×2 matrix where $ad - bc \neq 0$. Prove that A is nonsingular.

Hints: An example will not constitute a proof. One approach to a proof is to consider two general cases, $a = 0$ and $a \neq 0$. No matter what approach you choose, make sure you are never ever dividing by a variable quantity that could be zero.

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V-1 (Section VO)

Prove Property AAC of Theorem VSPCV. That is:

If $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{C}^m$, then $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$.

Write your own proof in the style of proofs of Property DSCA (Theorem VSPCV) and Property CC (Solution VO.T13).

Hints: Think carefully about the two types of equality you will likely use in a proof. A successful proof will likely conclude with an appeal to Definition VE.

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V-2 (Section LI)

Prove that the set of standard unit vectors (Definition SUV) is linearly independent.

Hints: There are several ways to do this, some easier than others. If you choose an approach that claims two matrices are equal, then you must establish that equality very carefully (how do we do that?). In other approaches, be very careful that the hypotheses of any theorems you use are satisfied.

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M-1 (Section MM)

Suppose that A is an $m \times n$ matrix with a row where every entry is zero. Suppose that B is an $n \times p$ matrix. Prove that AB has a row where every entry is zero.

Hints: Theorem EMP should be useful, and you want to be explicit about which row of A has the zeros and which row of AB has the zeros. Which row is all zeros? Does it have a name?

It is highly unlikely you can prove this without relying heavily on symbols and equations.

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M-2 (Section MM)

Use Theorem EMP to prove part (2) of Theorem MMIM: If A is an $m \times n$ matrix and I_m is the identity matrix of size m , then $I_m A = A$.

Hint: Carefully studying the proof of part (1) of Theorem MMIM would be a good way to start.

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VS-1 (Section S)

Give an example of using Theorem TSS by proving that $W = \left\{ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \middle| 5x_1 + 3x_2 - 8x_3 + 2x_4 = 0 \right\}$ is a subspace of \mathbb{C}^4 .

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VS-2 (Section PD)

Carefully read Exercise PD.T60 and its solution (Solution PD.T60). Prove the “more general” result given in the solution, using the book’s solution as a model.

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D&E-1 (Section DM)

Prove that the inverse of an elementary matrix is a single elementary matrix.

Hints: Seek inspiration from Exercise RREF.T10

If you are tempted to “cancel” a matrix, you may want to think again.

This theorem does not say an elementary matrix is invertible — it says the inverse of an elementary matrix has the form of an elementary matrix — just one. How would you convince somebody of that?

There are three different types of elementary matrices, yes?

A well-written argument will likely consider both the *function* and the *form* of an elementary matrix.

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D&E-2 (Section EE)

Suppose that A is a matrix that is equal to its inverse, $A = A^{-1}$. Prove that the only possible eigenvalues of A are $\lambda = 1$ and $\lambda = -1$. Give an example of matrix that is equal to its inverse and actually has both of these possible values as eigenvalues.

Hints: Theorem EIM says that if λ is an eigenvalue of A , then λ^{-1} is an eigenvalue of A^{-1} . The hypothesis that $A = A^{-1}$ does not allow you to *immediately* conclude that $\lambda = \lambda^{-1}$. Also, Theorem EIM makes no statements about eigenvectors. However, you might study the proof of Theorem EIM for useful techniques. If you are tempted to “cancel” a vector, you need a theorem to back it up.

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LT-1 (Section LT)

Prove that the function

$$T: \mathbb{C}^3 \rightarrow \mathbb{C}^2, \quad T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} 2x_1 - x_2 + 5x_3 \\ 3x_1 + 8x_3 \end{bmatrix}$$

is a linear transformation.

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LT-2 (Section IVLT)

Suppose $T: U \rightarrow V$ is a surjective linear transformation and $\dim(U) = \dim(V)$. Prove that T is an invertible linear transformation. Write your proof in a style that mimics proofs in the textbook.

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R-1 (Section MR)

Consider the two linear transformations,

$$T: M_{22} \rightarrow P_2, \quad T \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = (2a - b + 3c + d) + (2b - c + 2d)x + (4a - 2b + 3c + d)x^2$$

$$S: P_2 \rightarrow \mathbb{C}^2, \quad S(p + qx + rx^2) = \begin{bmatrix} 2p + q - 3r \\ 5p + 2q - 4r \end{bmatrix}$$

and the bases of M_{22} , P_2 and \mathbb{C}^2 (respectively)

$$B = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ 1 & -4 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}, \begin{bmatrix} -2 & -4 \\ -5 & 4 \end{bmatrix} \right\}$$
$$C = \{1 + x, -2 - 3x + x^2, -2 - 2x + x^2\}$$
$$D = \left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$$

Verify the conclusion of Theorem MRCLT. In other words, build the three matrix representations of T , S and $S \circ T$ individually and check that they are related by the matrix product as in the theorem.

- First due: **Monday, May 5**, *before* the start of class. Late submissions are not accepted. Failure to follow the directions below can result in an automatic retry with no comments.
- Use SageMath Cloud or TeXonWeb to make a PDF (links on course page). Do not trust these sites as a place to store your work. Keep copies of your source files so retries are easier.
- As a graded assignment, **this is your own work**.
(1) No discussions at all with any other students or anybody else. (2) All writing is your own. (3) FCLA source may be read for L^AT_EX help, but content may not be used. (4) Re-read in the Academic Handbook about Academic Integrity if you are unsure what any of this means.
- Include your name on each page. Do not print anything on the back side of any of page. **Staple** your work together, with this cover sheet on top, and each problem stapled individually.
- Examples are not proofs, and may not be used as justifications for general statements.
- Your audience is your peers in this course, not me. You may quote acronyms without providing any kind of reference. This means repeating whole theorems or definitions verbatim from the text is not necessary and is counter-productive.
- Write with complete sentences. You are **required** to use L^AT_EX as necessary. For example, **every** single symbol should be treated as math. So a matrix A would be surrounded by dollar signs. All vectors need to be bold (\mathbf{v}) or need to have arrows (\vec{v}). Alignment of equations will be enforced.
- Only use theorems and definitions from the relevant chapter, or earlier. This applies to retries.
- Retry: submit a complete, new version. Reassemble: cover sheet with my comments on top, **your current retry next**, then your previous work **in reverse chronological order**. A retry that makes no new progress, or that ignores substantial corrections or repeated hints, will become a No Pass.

R-2 (Section VR)

Let C be the crazy vector space from Section VS (Definition CVS). From Example DC, we know C has dimension 2. By Theorem CFDVS we can conclude that C must be isomorphic to \mathbb{C}^2 . Construct a function $T: \mathbb{C}^2 \rightarrow C$ that is a candidate for an isomorphism between these two vector spaces by giving an *explicit* formula for T . Then give a convincing argument that T is indeed an isomorphism.

DO NOT WRITE ON THIS SHEET