Math 290 Name: Dr. Beezer
Exam 4 Spring 2014
Chapter VS

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may compute reduced row-echelon form and matrix inverses with Sage, and provide this as justification for parts of your answers, so long as you explain your input and list the output in your solution.

1. The set $B = \left\{2x^2 + 2x + 1,\ 4x^2 + 5x + 2,\ -3x^2 - 4x - 2\right\}$ is a basis of the vector space, P_2, of polynomials with degree at most 2 (you may assume this). Write the vector $\mathbf{u} = x^2 + x + 1$ as a linear combination of the vectors in B. (15 points)

2. $S = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \middle|\, 3a + 2b = 0 \right\}$ is a subset of the vector space $\mathbb{C}^2$. Prove that S is a subspace of $\mathbb{C}^2$. (15 points)

3. The set $U = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \middle| \; a - 3b + 2c = 0 \right\}$ is a subspace of $\mathbb{C}^3$ (you may assume this). (40 points)

(a) Find a minimal spanning set for U.

(b) Prove that your set from part (a) is linearly independent.

(c) What is the dimension of U?

(d) Is $R = \left\{ \begin{bmatrix} -4 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix} \right\} \subseteq U$ a basis of U? Explain.

(e) Is $T = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ -1 \\ 1 \end{bmatrix} \right\} \subseteq U$ a basis of U? Explain.

4. Suppose that V is a vector space and $\mathbf{v} \in V$. Prove that $-\mathbf{v} = (-1)\mathbf{v}$. (15 points)

5. Suppose that V is a vector space and $\mathbf{v}_1, \mathbf{v}_2 \in V$. Prove that $X = \{\mathbf{v}_1, \mathbf{v}_2\}$ is a spanning set for V if and only if $Y = \{2\mathbf{v}_1 + 5\mathbf{v}_2, \mathbf{v}_1 + 3\mathbf{v}_2\}$ is a spanning set for V. (15 points)