Math 290 Name: Dr. Beezer
Exam 5 Spring 2014
Chapters D&E

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use computations from Sage as justification for your answers **only** as indicated in each question.

1. Compute the determinant of A (without using Sage). (15 points)

$$A = \begin{bmatrix} 2 & -1 & 4 & -1 \\ -2 & 2 & 0 & 2 \\ 4 & 3 & 0 & 3 \\ 0 & -1 & -3 & -1 \end{bmatrix}$$

2. Given the matrix B below, find an invertible matrix S such that $S^{-1}BS$ is a diagonal matrix. You may use Sage to compute eigenvalues and to row-reduce matrices. (15 points)

$$B = \begin{bmatrix} -4 & 12 & -18 \\ -6 & 23 & -36 \\ -3 & 12 & -19 \end{bmatrix}$$

3. Consider the square matrix A. (40 points)

$$A = \begin{bmatrix} -27 & 48 & 31 & -20 \\ -25 & 47 & 25 & -13 \\ 40 & -80 & -36 & 13 \\ 42 & -84 & -42 & 18 \end{bmatrix}$$

(a) Use Sage to compute a factored version of the characteristic polynomial.

(b) Without using Sage, determine the eigenvalues of A and their geometric multiplicities.

(c) Using only Sage's reduced row-echelon form method for computational assistance, determine all the eigenspaces of A along with their geometric multiplicities.

(d) Is A diagonalizable? Explain fully.

4. Suppose that A is an $n \times n$ matrix and $\lambda \in \mathbb{C}$. Define

$$U = \{\mathbf{x} \in \mathbb{C}^n | \, A\mathbf{x} = \lambda\mathbf{x}\}$$

Prove additive closure for U, which is one of the three conditions of checking that a set is a subspace. (15 points)

5. Suppose that A, B and C are three $n \times n$ matrices that are equal to each other, except for the entries in column k. In A column k is the vector $\mathbf{x}$, in B column k is the vector $\mathbf{y}$, and in C column k is the vector $\mathbf{x}+\mathbf{y}$. Prove that $\det(C) = \det(A) + \det(B)$. (15 points)

Pictorially: $A = [\ldots|\mathbf{x}|\ldots]$ $B = [\ldots|\mathbf{y}|\ldots]$ $C = [\ldots|\mathbf{x}+\mathbf{y}|\ldots]$