Math 290 Name: Key Dr. Beezer
Exam 1 Spring 2015
Chapter SLE

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use Sage to row-reduce matrices, except in the question that asks you to row-reduce without Sage. No other use of Sage may be used as justification for your answers.

1. Solve the following system of linear equations and express the solutions as a set of column vectors. (15 points)

$$\begin{aligned} 3x_1 + 8x_2 + 6x_3 &= 11 \\ 3x_2 - 4x_3 &= 3 \\ 2x_1 + 2x_2 + 3x_3 &= 4 \\ x_1 + 6x_2 - 2x_3 &= -2 \end{aligned}$$

Augmented matrix of the system $\left[\begin{array}{ccc|c} 3 & 8 & 6 & 11 \\ 0 & 3 & -4 & 3 \\ 2 & 2 & 3 & 4 \\ 1 & 6 & -2 & -2 \end{array}\right]$

$\xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ Last column is a pivot column, so by RCLS the system is inconsistent.

So solution set is $\{\ \} = \emptyset$.
(empty set)

2. Solve the following system of linear equations and express the solutions as a set of column vectors. (20 points)

$$\begin{aligned} x_1 - 2x_2 - 6x_3 - 6x_4 &= 5 \\ x_2 + 4x_3 + 5x_4 &= -5 \\ 2x_1 - x_2 + x_3 + 4x_4 &= -6 \end{aligned}$$

Augmented matrix of the system $\left[\begin{array}{cccc|c} 1 & -2 & -6 & -6 & 5 \\ 0 & 1 & 4 & 5 & -5 \\ 2 & -1 & 1 & 4 & -6 \end{array}\right]$

$\xrightarrow{\text{RREF}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -3 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & -1 \end{array}\right]$

RCLS $\Rightarrow$ consistent
FVCS $\Rightarrow$ $n - r = 4 - 3 = 1$ free variable (x_4)

Equations: $x_1 = -3 - 2x_4$
$x_2 = -1 - x_4$
$x_3 = -1 - x_4$

$$S = \left\{ \begin{bmatrix} -3-2x_4 \\ -1-x_4 \\ -1-x_4 \\ x_4 \end{bmatrix} \middle|\ x_4 \in \mathbb{C} \right\}$$

3. Without using Sage, find a matrix B in reduced row-echelon form which is row-equivalent to A. It is especially important to show all of your work, so it is clear you have not used Sage. (20 points)

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 \\ 2 & 1 & 5 & 2 \\ -3 & 3 & -3 & -3 \end{bmatrix} \xrightarrow[3R_1 + R_3]{-2R_1 + R_2} \begin{bmatrix} 1 & 1 & 3 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 6 & 6 & 0 \end{bmatrix} \xrightarrow{-1R_2} \begin{bmatrix} 1 & 1 & 3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 6 & 6 & 0 \end{bmatrix}$$

$$\xrightarrow[-6R_2 + R_3]{-R_2 + R_1} \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

B

4. Determine if the matrix below is nonsingular or singular. Explain your reasoning carefully and thoroughly. (15 points)

$$\begin{bmatrix} -4 & 5 & 5 & -5 & 2 & -7 \\ 4 & -3 & -3 & 2 & 3 & -2 \\ 4 & -5 & -4 & 3 & -1 & 2 \\ 1 & -1 & 0 & -1 & 1 & -4 \\ 2 & -1 & -1 & 0 & 3 & -4 \\ -5 & 6 & 6 & -6 & 2 & -8 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 0 & 2 & -2 \\ 0 & 1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

not I_6 so by Theorem NMRRI the original matrix is singular.

5. Say as much as possible about the solution set of each system, along with justifications for your answers. (15 points)

   (a) 12 variables, 9 equations.

   inconsistent
   OR
   Consistent & CMVEI implies infinitely many solutions.

   (b) Coefficient matrix is nonsingular.

   Theorem NMUS says there will be a unique solution.

   (c) Homogeneous, 8 variables and 8 equations.

   $\mathbf{0}$ is a solution by Theorem HSC.
   We cannot say more, as we do not know if the coefficient matrix is singular or nonsingular.

6. Suppose $\mathcal{LS}(A, \mathbf{b})$ is a system of equations with solution set $\{\mathbf{0}\}$. Give a careful, well-written, proof that the system is homogeneous. (15 points)

n variables, m equations. $x_1 = x_2 = \cdots = x_n = 0$ is a solution

Equation i (for $1 \le i \le m$):

$$\begin{aligned} b_i &= a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \\ &= a_{i1}(0) + a_{i2}(0) + \cdots + a_{in}(0) \leftarrow \text{evaluate with solution} \\ &= 0 + 0 + \cdots + 0 = 0 \end{aligned}$$

So $b_i = 0$ for <u>all</u> $1 \le i \le m$. This is the definition of a homogeneous system.