Math 290 Name: Dr. Beezer
Exam 6 Spring 2015
Chapter LT

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use Sage to manipulate and row-reduce matrices. Be sure to make it clear what you have input to Sage, and show any output you use to justify your answers.

1. Consider the following questions about the linear transformation T. (30 points)

$$T\colon \mathbb{C}^2 \to \mathbb{C}^3, \qquad T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} 2a+b \\ -2a-b \\ 4a+2b \end{bmatrix}$$

(a) Compute the kernel of T, $\mathcal{K}(T)$.

(b) Based on your answer to the previous question, is T injective?

(c) Find two vectors $\mathbf{x}$ and $\mathbf{y}$ such that $T(\mathbf{x}) = T(\mathbf{y})$.

(d) Compute the dimension of the range of T, $\dim(\mathcal{R}(T))$.

(e) Based on your answer to the previous question, is T surjective?

(f) Find a vector $\mathbf{x}$ whose preimage, $T^{-1}(\mathbf{x})$, is empty.

2. Given $R\colon \mathbb{C}^3 \to P_2$ below, find an explicit formula for values of R^{-1}. You may assume that R is invertible. (P_2 is the vector space of polynomials with degree at most 2.) (20 points)

$$R\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = (3a + b - 6c) + (2a + b - 5c)\,x + (-3a + 2b - 2c)\,x^2$$

3. Verify that the function below is a linear transformation. (20 points)

$$S\colon \mathbb{C}^3 \to \mathbb{C}^2,\ S\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a - 2b \\ b + 4c \end{bmatrix}$$

4. Suppose that $T\colon U \to W$ is a linear transformation such that $T(\alpha\mathbf{u}) = \alpha^2 T(\mathbf{u})$ for all $\alpha \in \mathbb{C}$ and all $\mathbf{u} \in U$. Prove that $T(\mathbf{u}) = \mathbf{0}$ for all $\mathbf{u} \in U$. (15 points)

5. Suppose that $T\colon V \to W$ is an injective linear transformation and that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \dots, \mathbf{v}_m\}$ is a linearly independent subset of V. Prove that $R = \{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3), \dots, T(\mathbf{v}_m)\}$ is a linearly independent subset of W. (15 points)