Math 290 Name: Dr. Beezer
Exam 7 Spring 2015
Chapter R

Show *all* of your work and *explain* your answers fully. There is a total of 90 possible points.
You may use Sage to manipulate matrices and vectors, and compute reduced row-echelon form, inverses, determinants and eigen-stuff. Be sure to make it clear what you have input to Sage, and show any output you use to justify your answers. $\mathbb{C}^n$ is the vector space of column vectors with n entries, P_n is the vector space of polynomials with degree at most n and M_{mn} is the vector space of $m \times n$ matrices.

1. Compute the matrix representation of T relative to the bases B and C, $M^T_{B,C}$. (15 points)

$$T\colon P_1 \to M_{12}, \qquad T\left(a+bx\right) = \begin{bmatrix} 2a+b & a-b \end{bmatrix}$$

$$B = \{1+2x, 3-x\} \qquad C = \left\{\begin{bmatrix} 1 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 5 \end{bmatrix}\right\}$$

2. Use vector representations to efficiently answer the following questions. (15 points)

   (a) Is $S = \left\{1-4x+8x^2, 1-3x+6x^2, -1+4x-7x^2\right\}$ a linearly independent set in P_2?

   (b) Does the set $Q = \left\{-7-3x+x^2, -5-2x+x^2, -3-x+x^2\right\}$ span P_2?

3. Use a matrix representation for the following questions about the linear transformation T. (30 points)

$$T\colon M_{22} \to P_2, \qquad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (-7a - 5b + 10c - 31d) + (-2a - b + 2c - 8d)\,x + (-3a - 2b + 4c - 13d)\,x^2$$

(a) Compute the kernel of T, $\mathcal{K}(T)$.

(b) Based on your answer to the previous question, is T injective?

(c) Find two vectors $\mathbf{x}$ and $\mathbf{y}$ such that $T(\mathbf{x}) = T(\mathbf{y})$.

(d) Compute the dimension of the range of T, $\dim(\mathcal{R}(T))$.

(e) Based on your answer to the previous question, is T surjective?

(f) Find a vector $\mathbf{x}$ whose preimage, $T^{-1}(\mathbf{x})$, is empty.

4. Determine a basis B for P_2 so that the matrix representation of S relative to B is a diagonal matrix. (15 points)

$$S\colon P_2 \to P_2, \qquad S\left(a + bx + cx^2\right) = (-23a + 12b + 6c) + (-48a + 25b + 12c)\,x + (12a - 6b - 2c)\,x^2$$

5. Compute an explicit formula for L^{-1}. (You may assume L is invertible.) (15 points)

$$L\colon \mathbb{C}^3 \to P_2, \qquad L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = (3a - b + 3c) + (4a - b + 3c)\,x + (4a + 2b - 7c)\,x^2$$