Math 290   Name: Dr. Beezer
Exam 1   Spring 2016
Chapter SLE

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.

For computational problems, place your answer in the provided boxes. Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices, except in the question that asks you to row-reduce without Sage. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Solve the following system of linear equations and express the solutions as a set of column vectors. (15 points)

$$x_1 + 2x_2 + x_3 - x_4 = 2$$
$$2x_1 - x_2 + 2x_3 + x_4 = 0$$
$$4x_1 + 3x_2 + 4x_3 - x_4 = 3$$

Answer:

2. Solve the following system of linear equations and express the solutions as a set of column vectors. (15 points)

$$-4x_1 + 5x_2 - 7x_3 + 2x_4 = -10$$
$$-5x_1 + 6x_2 - 8x_3 + 4x_4 = -8$$
$$-5x_1 + 6x_2 - 7x_3 + 7x_4 = 0$$

Answer:

3. Without using Sage, find a matrix B in reduced row-echelon form which is row-equivalent to A. It is especially important to show all of your work, so it is clear you have not used Sage. (20 points)

$$A = \begin{bmatrix} 1 & 3 & -2 & 4 \\ 0 & 0 & 1 & -1 \\ 2 & 6 & -2 & 10 \end{bmatrix}$$

Answer:

4. Determine if the matrix below is nonsingular or singular. Explain your reasoning carefully and thoroughly. (15 points)

$$\begin{bmatrix} -2 & -4 & -4 & 0 & -3 & -7 & -2 \\ 1 & 0 & 3 & 5 & -4 & -2 & -5 \\ -1 & 2 & 0 & -4 & 4 & 4 & 7 \\ -1 & -1 & -5 & -6 & 4 & 1 & 3 \\ 1 & 1 & 2 & 2 & -1 & 1 & -2 \\ 0 & 4 & 5 & -2 & 4 & 6 & 7 \end{bmatrix}$$

5. Say as much as possible about the solution set of each system, along with justifications for your answers. (15 points)

   (a) Coefficient matrix is singular.

   (b) 13 variables, 5 equations.

   (c) Homogeneous, 8 variables and 7 equations.

6. Suppose that a homogeneous system has the same number of equations as variables, and that two of the equations are identical. Prove that the system has infinitely many solutions. (15 points)