Math 290 Name: Key Dr. Beezer
Exam 1 Spring 2016
Chapter SLE

Show *all* of your work and *explain* your answers fully. There is a total of ~~100~~ 95 possible points.
For computational problems, place your answer in the provided boxes. Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices, except in the question that asks you to row-reduce without Sage. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Solve the following system of linear equations and express the solutions as a set of column vectors. (15 points)

$$\begin{aligned} x_1 + 2\,x_2 + x_3 - x_4 &= 2 \\ 2\,x_1 - x_2 + 2\,x_3 + x_4 &= 0 \\ 4\,x_1 + 3\,x_2 + 4\,x_3 - x_4 &= 3 \end{aligned}$$

Augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 2 & -1 & 2 & 1 & 0 \\ 4 & 3 & 4 & -1 & 3 \end{array}\right] \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 1 & 1/5 & 0 \\ 0 & 1 & 0 & -3/5 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

last column is a pivot column, so by RCLS, no solutions

Answer:

$\{\ \}$ or $\emptyset$

2. Solve the following system of linear equations and express the solutions as a set of column vectors. (15 points)

$$\begin{aligned} -4\,x_1 + 5\,x_2 - 7\,x_3 + 2\,x_4 &= -10 \\ -5\,x_1 + 6\,x_2 - 8\,x_3 + 4\,x_4 &= -8 \\ -5\,x_1 + 6\,x_2 - 7\,x_3 + 7\,x_4 &= 0 \end{aligned}$$

Augmented matrix

$$\left[\begin{array}{cccc|c} -4 & 5 & -7 & 2 & -10 \\ -5 & 6 & -8 & 4 & -8 \\ -5 & 6 & -7 & 7 & 0 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -4 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 0 & 1 & 3 & 8 \end{array}\right]$$

$D = \{1, 2, 3\}$, $F = \{4\}$, $r = 3$
Consistent

$x_1 = -4 + 2x_4$
$x_2 = 6 - 3x_4$
$x_3 = 8 - 3x_4$

Answer:

$$\left\{ \begin{bmatrix} -4+2x_4 \\ 6-3x_4 \\ 8-3x_4 \\ x_4 \end{bmatrix} \;\middle|\; x_4 \in \mathbb{C} \right\}$$

3. Without using Sage, find a matrix B in reduced row-echelon form which is row-equivalent to A. It is especially important to show all of your work, so it is clear you have not used Sage. (20 points)

$$A = \begin{bmatrix} 1 & 3 & -2 & 4 \\ 0 & 0 & 1 & -1 \\ 2 & 6 & -2 & 10 \end{bmatrix}$$

$$\xrightarrow{-2R_1+R_3} \begin{bmatrix} 1 & 3 & -2 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

$$\xrightarrow[-2R_2+R_3]{2R_2+R_1} \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{2}R_3} \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow[R_3+R_2]{-2R_3+R_1} \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Determine if the matrix below is nonsingular or singular. Explain your reasoning carefully and thoroughly. (15 points)

$$\begin{bmatrix} -2 & -4 & -4 & 0 & -3 & -7 & -2 \\ 1 & 0 & 3 & 5 & -4 & -2 & -5 \\ -1 & 2 & 0 & -4 & 4 & 4 & 7 \\ -1 & -1 & -5 & -6 & 4 & 1 & 3 \\ 1 & 1 & 2 & 2 & -1 & 1 & -2 \\ 0 & 4 & 5 & -2 & 4 & 6 & 7 \end{bmatrix} \xrightarrow[\text{(Sage)}]{\text{RREF}} I_7, \text{ the } 7\times 7 \text{ identity matrix.}$$

added row 7 during exam

By theorem NMRRI, we see that the matrix is nonsingular.

5. Say as much as possible about the solution set of each system, along with justifications for your answers. (15 points)

   (a) Coefficient matrix is singular.

   A unique solution is impossible by Theorem NMUS.
   Thus, by Theorem PSSLS, there is no solution,
   or infinitely many.

   (b) 13 variables, 5 equations.

   Inconsistent, or
   by Theorem CMVEI there are infinitely many solutions.

   (c) Homogeneous, 8 variables and 7 equations.

   By Theorem HMVEI there are infinitely many solutions.

6. Suppose that a homogeneous system has the same number of equations as variables, and that two of the equations are identical. Prove that the system has infinitely many solutions. (15 points)

Form the coefficient matrix and row-reduce,
The identical equations make identical rows and then
row operations will make a zero row. Thus $r < n$.
Homogenous systems are always consistent (Theorem HSC).
So by theorem CSRN the system has
infinitely many solutions.