Math 290 Name: Dr. Beezer
Exam 4 Spring 2016
Chapter VS

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
For computational problems, place your answer in the provided boxes. Partial credit is proportional to the quality of your explanation. You may use Sage to form, manipulate and row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features of something I can't see).

1. For the matrix A below, compute the dimensions of the null space, the column space, the row space, and the left null space. (15 points)

$$A = \begin{bmatrix} -1 & 4 & 0 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -2 & 5 & 4 & -3 \\ -2 & 8 & 1 & -3 & 3 & 7 & -8 \\ 2 & -8 & 0 & 3 & -2 & -5 & 7 \end{bmatrix}$$

Answer:

2. Compute a basis for the subspace W of the vector space of 2×2 matrices, M_{22}. (15 points)

$$W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \, 2a + 5b + 13c - d = 0, a + 3b + 8c - d = 0 \right\}$$

Answer:

3. The questions on this page ask about properties of subsets of the vector space of polynomials with degree at most 2, P_2. Give complete and detailed explanations of your answers. (40 points)

(a) $Q = \{3x^2 - 6x + 2,\ -4x^2 + x + 9\}$. Does Q span P_2?

(b) $R = \{-x^2 + 2x + 3,\ 4x^2 + 2x - 4,\ x^2 + 5x - 1,\ 3x^2 + 2x - 5\}$. Is R linearly independent in P_2?

(c) $S = \{x^2 - x + 2,\ -3x^2 + 6x - 8,\ 3x^2 - 5x + 7,\ -4x^2 + 4x - 7\}$. Does S span P_2?

(d) $T = \{2x^2 - x + 1,\ -3x^2 + 1,\ x^2 - 2x + 3,\ -5x^2 + x\}$. Does T span P_2?

4. Suppose that V is a vector space. Prove that $-\mathbf{v} = (-1)\mathbf{v}$ for each $\mathbf{v} \in V$. (15 points)

5. Consider the subsets S and T from P_2, the vector space of polynomials with degree at most 2. Prove that their spans are equal, that is $\langle S \rangle = \langle T \rangle$. (15 points)

$S = \{1 + x + x^2,\ -2 + x - x^2\}$ $\qquad$ $T = \{-5 + 7x - x^2,\ -4 + 5x - x^2\}$