Math 290 Name: Dr. Beezer
Exam 1 Spring 2017
Chapter SLE

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
For computational problems, place your answer in the provided boxes. Partial credit is proportional to the quality of your explanation. You may use Sage to manipulate and row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Is the vector $\mathbf{x}$ an element of the span of $S$, $\langle S\rangle$? Explain carefully why, or why not. (15 points)

$$\mathbf{x} = \begin{bmatrix} 1 \\ -2 \\ 2 \\ 0 \end{bmatrix} \qquad S = \left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ 1 \\ 1 \end{bmatrix} \right\}$$

2. Write a nontrivial relation of linear dependence on $T$, or explain why no such thing exists. (15 points)

$$T = \left\{ \begin{bmatrix} -3 \\ -1 \\ 2 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ -4 \\ -3 \\ 6 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ 5 \\ -3 \\ -4 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ -2 \\ -8 \\ 7 \end{bmatrix} \right\}$$

Answer:

3. Use the appropriate theorem to find a set $P$ that is (1) a subset of $S$, (2) linearly independent, and (3) the span of $P$ equals the span of $S$, $\langle P\rangle = \langle S\rangle$. (20 points)

$$S = \left\{ \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} -8 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix} \right\}$$

Answer:

4. Given the matrix $A$, use the appropriate theorem to find a linearly independent set $R$ so that the span of $R$ is the null space of $A$, $\langle R\rangle = \mathcal{N}(A)$. (20 points)

$$A = \begin{bmatrix} -1 & 3 & -3 & 0 \\ 2 & -7 & 6 & -1 \\ -1 & 5 & -3 & 2 \\ 0 & -5 & 0 & -5 \\ -2 & 5 & -6 & -1 \end{bmatrix}$$

Answer:

5. Suppose that $\mathbf{u}, \mathbf{v} \in \mathbb{C}^m$. Give a proof that $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ using a style that finishes with an application of Theorem CVE. (15 points)

6. Suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a set of orthogonal vectors from $\mathbb{C}^m$. Prove that $\mathbf{v}_1$ is orthogonal to $2\mathbf{v}_2 + 5\mathbf{v}_3$. (15 points)