

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points. Partial credit is proportional to the quality of your explanation. Available Sage commands, or prohibitions, are present in each question. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Compute the determinant of  $B$ ,  $\det(B)$ , without any use of Sage. (10 points)

$$B = \begin{bmatrix} 3 & 1 & 2 \\ -1 & 0 & 3 \\ 2 & 2 & 5 \end{bmatrix}$$

2. Compute the eigenvalues of  $C$ , without any use of Sage. (15 points)

$$C = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$$



3. Answer the following questions about the matrix  $A$ . (45 points)

$$A = \begin{bmatrix} -10 & -3 & 27 & -3 & -39 & -24 \\ -30 & -37 & 99 & 87 & -3 & -60 \\ -12 & -9 & 35 & 15 & -21 & -24 \\ -3 & -9 & 15 & 26 & 15 & -6 \\ -3 & 0 & 6 & -3 & -10 & -6 \\ 0 & -3 & 3 & 9 & 9 & 2 \end{bmatrix}$$

- (a) Use Sage to compute the eigenvalues of  $A$ . You may use any Sage commands you like.
- (b) Compute the algebraic and geometric multiplicity of each eigenvalue of  $A$ . You may use any Sage commands you like.
- (c) Compute a basis for the eigenspace of one of the eigenvalues, using Sage only to manipulate and row-reduce matrices.
- (d) Illustrate the use of a theorem (not Sage) to determine that  $A$  is diagonalizable.
- (e) With no further use of Sage, use your computations above to provide a diagonal matrix that is similar to  $A$ .
- (f) With no further use of Sage, compute (by hand) the determinant of  $A$ .



4. Similarity of square matrices is an equivalence relation. For this relation, prove the “transitive” part of the three-part definition of an equivalence relation (15 points)

5. Suppose that  $A$  is a nonsingular matrix and  $\lambda$  is an eigenvalue of  $A$ . Prove that  $1/\lambda$  is an eigenvalue of  $A^{-1}$ . (15 points)

