Math 290 Name: Dr. Beezer
Exam 1 Fall 2019
Chapter SLE

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices, except in the question that asks you to row-reduce without Sage. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Solve the following system of linear equations and express the solutions as a set of column vectors. (15 points)

$$\begin{aligned}
3x_1 + x_2 + 7x_3 + 3x_4 &= 13 \\
-x_1 \qquad - 4x_3 + x_4 &= 3 \\
-2x_1 - x_2 - 2x_3 - 5x_4 &= -20 \\
x_1 + x_2 - 3x_3 + 8x_4 &= 31
\end{aligned}$$

2. Without using Sage, find a matrix B in reduced row-echelon form which is row-equivalent to A. It is especially important to show all of your work, so it is clear you have not used Sage. (15 points)

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & 1 & 2 \\ 2 & 3 & 7 \end{bmatrix}$$

3. Is the matrix B singular or not? Provide a justification for your answer. (15 points)

$$B = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 3 & -2 & 8 & 1 \\ -1 & 0 & 1 & -6 \\ 2 & -1 & 8 & 0 \end{bmatrix}$$

4. Determine the solutions to the homogeneous system $\mathcal{LS}(B, \mathbf{0})$ and express them compactly as a (infinite) set of column vectors. Then do the necessary algebra to convince someone that your solution set is correct. (15 points)

$$B = \begin{bmatrix} -1 & -2 & 1 & -2 \\ -2 & -5 & 3 & -8 \\ 1 & 2 & 0 & -1 \end{bmatrix}$$

5. Suppose that A is a nonsingular 3×3 matrix, and C is a singular 3×3 matrix. Let $\mathbf{0}$ and $\mathbf{c}$ be the vectors below. For each of the four linear systems, say as much as you can about the solution set. (20 points)

$$\mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} -1 \\ 3 \\ -4 \end{bmatrix}$$

(a) $\mathcal{LS}(A, \mathbf{0})$

(b) $\mathcal{LS}(A, \mathbf{b})$

(c) $\mathcal{LS}(C, \mathbf{0})$

(d) $\mathcal{LS}(C, \mathbf{b})$

6. Suppose we have a homogeneous linear system that has coefficient matrix A, $\mathcal{LS}(A, \mathbf{0})$. Suppose further that every column of A is the same (i.e. identical). Prove that the system has infinitely many solutions. (15 points)