Math 290 Name: Dr. Beezer
Exam 2 Spring 2020
Chapter V

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Determine if the vector $\mathbf{x}$ is in the span of S, $\mathbf{x} \in \langle S \rangle$, with a complete and careful explanation. (15 points)

$$\mathbf{x} = \begin{bmatrix} -3 \\ 1 \\ 2 \\ 1 \end{bmatrix} \qquad S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 2 \\ -3 \end{bmatrix} \right\}$$

2. Determine if the set T is linearly independent, with a complete and careful explanation. (15 points)

$$T = \left\{ \begin{bmatrix} -1 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ -8 \\ 6 \\ -6 \end{bmatrix} \right\}$$

3. Find a linearly independent set R that spans the null space of A, that is, $\mathcal{N}(A) = \langle R \rangle$. (20 points)

$$A = \begin{bmatrix} 0 & 1 & 0 & 2 & -1 \\ -1 & 2 & -2 & 7 & 5 \\ -1 & 2 & -1 & 6 & 1 \\ 0 & 2 & -2 & 6 & 6 \end{bmatrix}$$

4. Find a linearly independent set K whose span is equal to the span of L, $\langle K \rangle = \langle L \rangle$. (20 points)

$$L = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ -6 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ -6 \end{bmatrix}, \begin{bmatrix} -5 \\ -1 \\ 4 \end{bmatrix} \right\}$$

5. Provide a proof for Property DSAC: if α, $\beta \in \mathbb{C}$ and $\mathbf{u} \in \mathbb{C}^m$, then $(\alpha + \beta)\mathbf{u} = \alpha\mathbf{u} + \beta\mathbf{u}$. (15 points)

6. Suppose that S is a set of vectors, $S \subseteq \mathbb{C}^m$. Prove that if $\mathbf{x}$, $\mathbf{y} \in \langle S \rangle$, then $\mathbf{x} + \mathbf{y} \in \langle S \rangle$. (15 points)