Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Use the inverse of a matrix to find the solution set for the linear system $A\mathbf{x} = \mathbf{b}$. No credit will be given for other methods, so be sure to explain fully. (15 points)

$$A = \begin{bmatrix} -2 & 4 & -3 \\ 4 & -7 & 5 \\ -3 & 6 & -5 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

2. Find a non-zero vector $\mathbf{c}$ so that $A\mathbf{x} = \mathbf{c}$ is a consistent system, and a vector $\mathbf{d}$ so that $A\mathbf{x} = \mathbf{d}$ is an inconsistent system. In each case, *without actually attempting to solve a system of equations*, explain clearly how you know your answer is correct. So, for example, "guess and check" is not an explanation. Hint: a particular version of the column space of A could prove useful. (20 points)

$$A = \begin{bmatrix} 1 & 4 & -1 & 2 \\ -1 & -3 & 1 & -2 \\ 1 & 2 & -1 & 2 \end{bmatrix}$$

3. In each part of this question, find a set S of column vectors so that the span of S equals the column space of the matrix A (that is, $\mathcal{C}(A) = \langle S \rangle$) and S satisfies the additional requirements given in each part. Provide clear explanations on why your answer meets all the requirements. (25 points)

$$A = \begin{bmatrix} 87 & -348 & -124 & -11 & -633 & 50 \\ -35 & 140 & 50 & 4 & 255 & -20 \\ -12 & 48 & 17 & 2 & 87 & -7 \\ -7 & 28 & 10 & 1 & 51 & -4 \end{bmatrix}$$

(a) S is linearly independent and is a subset of the set of columns of A.

(b) S is linearly independent and is computed using results about row spaces.

(c) S is linearly independent and is computed using the matrix L from the extended echelon form of A.

4. Using the same matrix A from the previous question, find a set S of column vectors so that the span of S equals the row space of the matrix A (that is, $\mathcal{R}(A) = \langle S \rangle$) and S is a linearly independent set. Provide a clear explanation on why your answer meets all the requirements. (10 points)

5. Suppose that A is an $m \times n$ coefficient matrix of a homogeneous system where columns i and j are identical $(1 \leq i < j \leq n)$. Without analyzing the reduced row-echelon form of the coefficient matrix, prove that the system has infinitely many solutions. (15 points)

6. Suppose that A is an $m \times n$ matrix, and B and C are both $n \times p$ matrices. Prove the conclusion of Theorem MMDAA: $A(B + C) = AB + AC$. (15 points)