Math 290 Name: Dr. Beezer
Exam 4 Spring 2020
Chapter VS

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. The set S below is from the vector space P_3, polynomials with degree at most 3. Determine, with explanation, if S is linearly independent. (15 points)

   $S = \left\{x^3 + 3x^2 + 2x - 1,\, x^3 + 2x^2 + x - 2,\, x^3 + 4x^2 + 3x\right\}$

2. The set T below is from the vector space M_{22}, comprised of 2×2 matrices. Determine, with explanation, if T spans M_{22}. (15 points)

   $T = \left\{\begin{bmatrix}-5 & 3\\-3 & 3\end{bmatrix}, \begin{bmatrix}1 & -2\\1 & -3\end{bmatrix}, \begin{bmatrix}4 & -4\\3 & -5\end{bmatrix}, \begin{bmatrix}0 & 1\\-1 & 2\end{bmatrix}\right\}$

3. Determine the dimension of the subspace W of the vector space M_{22}, comprised of 2×2 matrices. (20 points)

$$W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \, a - 2c + d = 0, \; b + 4c - 3d = 0 \right\} \subseteq M_{22}$$

4. Demonstrate the use of the three parts of Theorem TSS to prove that U is a subspace of $\mathbb{C}^2$. (20 points)

$$U = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \middle| \, 2a - b = 0 \right\} \subseteq \mathbb{C}^2$$

5. Suppose that S is a finite set of vectors from the vector space V. Prove the third part of Theorem TSS showing that the span of S has scalar multiplication closure. In other words, show that if $\alpha \in \mathbb{C}$ and $\mathbf{x} \in \langle S \rangle$ then $\alpha\mathbf{x} \in \langle S \rangle$, without employing the theorem that says the span is a subspace. (15 points)

6. Suppose $B = \{\mathbf{v_1}, \mathbf{v_2}\}$ is a basis of the vector space V. Prove that $C = \{\mathbf{v_1} + \mathbf{v_2}, 2\mathbf{v_1} + \mathbf{v_2}\}$ is also a basis of V. (15 points)