

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points. Partial credit is proportional to the quality of your explanation. You may use Sage as allowed in the statement of a question. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Compute the determinant of  $A$ , without using Sage (so be sure to show all your work). (15 points)

$$A = \begin{bmatrix} 0 & 1 & 5 & 5 \\ 3 & 0 & 0 & 2 \\ 2 & 3 & 8 & 8 \\ 1 & 2 & -1 & -1 \end{bmatrix}$$

2. Compute the eigenvalues, eigenspaces, algebraic multiplicities, and geometric multiplicities of  $B$ . You may use Sage to obtain a factored characteristic polynomial and to row-reduce matrices. (20 points)

$$B = \begin{bmatrix} 7 & 12 & -36 \\ -8 & -15 & 48 \\ -2 & -4 & 13 \end{bmatrix}$$



3. Determine if the matrix in each part can be diagonalized. If the matrix cannot be diagonalized, give an explanation demonstrating the application of a theorem. When the matrix can be diagonalized, find a nonsingular matrix and a diagonal matrix so that a similarity transformation by the nonsingular matrix will produce the diagonal matrix. You may use Sage to obtain a factored characteristic polynomial and to row-reduce matrices. (35 points)

$$(a) C = \begin{bmatrix} 5 & 3 & 9 & -36 \\ 12 & 14 & -9 & -54 \\ 6 & 6 & -1 & -30 \\ 3 & 3 & 0 & -16 \end{bmatrix}$$

$$(b) E = \begin{bmatrix} -16 & -54 & 18 & -36 \\ 11 & 31 & -7 & 17 \\ 2 & 2 & 4 & -1 \\ -7 & -25 & 11 & -17 \end{bmatrix}$$



4. Suppose that  $\mathbf{x}$  and  $\mathbf{y}$  are eigenvectors of the matrix  $A$  for the eigenvalue  $\lambda$ . Suppose  $\alpha, \beta \in \mathbb{C}$  are such that  $\alpha\mathbf{x} + \beta\mathbf{y} \neq \mathbf{0}$ . Prove that then  $\alpha\mathbf{x} + \beta\mathbf{y}$  is an eigenvector of  $A$ . (15 points)

5. Suppose that  $A$  is a matrix similar to  $B = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ . Determine, with proof, a matrix similar to  $A^5$ . (15 points)

