Introductory Multi-Linear Algebra:

Dual Spaces to Tensor Products

Hayden Borg

University of Puget Sound

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A Naive Introduction

Tensors are a generalization of vectors

- n components per basis tensor
- Rank n
- Independent of the choice of basis
- Transform linearly

Examples

- Scalars: Tensors rank 0
- Vectors: Tensors Rank 1
- Matrix: Tensor Rank 2

A Naive Introduction: Stress Tensor

Consider the stress in a 3-dimension solid

- Convenient to represent cross-sectional area with a vector orthogonal to its surface
- There are three cross-sectional area for every point x, y, and z.
- For each cross-section, a force is described in three directions

A Naive Introduction: Stress Tensor

This information is most naturally encoded in a 3×3 array:

$$\begin{bmatrix} F_{xx} & F_{xy} & F_{xz} \\ F_{yx} & F_{yx} & F_{yz} \\ F_{zx} & F_{zy} & F_{zz} \end{bmatrix}$$