Math 290A Thursday, March 6 Section LISS

V vector space, set (vectors), vector addition & scalar multiplication

"10 good things" + theorems

Then linear combinations

Spans ① S = set of vectors

$\langle S \rangle$ = all linear combos of vectors in S (subspace)

~~HAG~~

② $W \subseteq V$ subspace of V

Find S so that $\langle S \rangle = W$

Ex P_2 - all polynomials w/ degree 2 or less

$R = \{1-x+2x^2, 1+3x^2, 1-x+3x^2\}$ spans P_2

Defn SE

$\langle R\rangle \subseteq P_2$ (EZ)

$P_2 \subseteq \langle R\rangle$ Grab $a+bx+cx^2 \in P_2$, in $\langle R\rangle$?

$$a+bx+cx^2 = a_1(1-x+2x^2) + a_2(1+3x^2) + a_3(1-x+3x^2)$$
$$= (a_1+a_2+a_3) + (-a_1-a_3)x + (2a_1+3a_2+3a_3)x^2$$

$$\begin{aligned} a_1 + a_2 + a_3 &= a \\ -a_1 \qquad - a_3 &= b \\ 2a_1 + 3a_2 + 3a_3 &= c \end{aligned} \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3a-c \\ 0 & 1 & 0 & a+b \\ 0 & 0 & 1 & -3a-b+c \end{array}\right]$$

(unique) solution for any choice of a, b, c

So yes, $P_2 \subseteq \langle R\rangle$ & so $\langle R\rangle = P_2$

Say "R spans P_2"; "R is a spanning set for P_2"

$\underline{Ex}$ $W = \{a+bx+cx^2 \mid a-4b+3c=0\} \subseteq P_2$ $\underline{\text{is}}$ a subspace of P_2.

Find a spanning set for W.

$[\,1\ \ -4\ \ 3\,]$ a dependent

b,c free

$W = \{a+bx+cx^2 \mid a = 4b-3c\}$

$= \{(4b-3c)+bx+cx^2 \mid b,c \in \mathbb{C}\}$

$= \{(4b+bx)+(-3c+cx^2) \mid b,c \in \mathbb{C}\}$

$= \{b(4+x)+c(-3+x^2) \mid b,c \in \mathbb{C}\}$

$= \langle \{4+x, -3+x^2\} \rangle$

So $T = \{4+x, -3+x^2\}$ is a spanning set for W.

[Bonus: T is linearly independent]