Math 290A Friday March 27 Problem session

- Mon B RQ GAM
- Tue D
- Wed Exam M (usual time)
- Thu PD
- Fri Problem Session, Writing Due

Office Hours

By appointment

LISS. T40 $C = \{\underset{\sim}{u}_1, \underset{\sim}{u}_2, \ldots, \underset{\sim}{u}_p\}$ lin ind, spans $\mathbb{C}^n$

A, B $m \times n$ matrices $A\underset{\sim}{u}_i = B\underset{\sim}{u}_i$ for $1 \le i \le n$

$\Rightarrow A = B$

$\underset{\sim}{e}_i$ = column i of I_n

$\underset{\sim}{e}_i = a_1\underset{\sim}{u}_1 + a_2\underset{\sim}{u}_2 + \cdots + a_p\underset{\sim}{u}_p$ (uniquely) Theorem VRRB

$(\in \mathbb{C}^n)$

S.T20

$A = \begin{bmatrix} x & x & x & x \\ & x & x & x \\ & & x & x \\ \mathbf{0} & & & x \end{bmatrix}$ upper triangular $[A]_{ij} = 0$ if $i > j$

$UT_n =$ all $n \times n$ u-t matrices Subspace of M_{nn}

Theorem TSS

(1) $UT_n \neq \emptyset$? $\mathbf{0} \in M_{nn}$ $[\mathbf{0}]_{ij} = 0$ for all i, j

So $[\mathbf{0}]_{ij} = 0$ if $i > j$. So $\mathbf{0} \in UT_n$

(2) Grab $A, B \in UT_n$. Know $[A]_{ij} = 0$ if $i > j$ & $[B]_{ij} = 0$ if $i > j$.

Look at $[A+B]_{ij}$ for $i > j$. Then

$$
\begin{aligned}
[A+B]_{ij} &= [A]_{ij} + [B]_{ij} && \text{Defn MA} \\
&= 0 + 0 && A, B \in UT_n \\
&= 0
\end{aligned}
$$

So $A + B \in UT_n$

$$\textcircled{2}\ A\underset{\sim}{e_i} = [\underset{\sim}{A_1} | \underset{\sim}{A_2} | \cdots | \underset{\sim}{A_n}]\underset{\sim}{e_i} = \underset{\sim}{A_i}\ \textcircled{1} \qquad \text{Similarly, } \textcircled{5}\ B\underset{\sim}{e_i} = \underset{\sim}{B_i}\ \textcircled{6}$$

$$\textcircled{3}\ A\underset{\sim}{e_i} = A(a_1\underset{\sim}{u_1} + \cdots + a_p\underset{\sim}{u_p}) \underset{\text{MMDAA}}{=} a_1A\underset{\sim}{u_1} + a_2A\underset{\sim}{u_2} + \cdots + a_pA\underset{\sim}{u_p}$$

$$= a_1B\underset{\sim}{u_1} + a_2B\underset{\sim}{u_2} + \cdots + a_pB\underset{\sim}{u_p} \quad \underline{\text{Hypothesis}}$$

$$= B(a_1\underset{\sim}{u_1} + a_2\underset{\sim}{u_2} + \cdots + a_p\underset{\sim}{u_p}) \quad \text{MMDAA}$$

$$= B\underset{\sim}{e_i}\ \textcircled{4}$$

$$\Rightarrow \underset{\sim}{A_i} = \underset{\sim}{B_i} \quad \text{for } 1 \le i \le n$$

So A & B have identical columns so $A = B$