Ex $W = \left\langle \left\{ \begin{bmatrix} -4 \\ \boxed{-3} \\ 3 \\ -11 \end{bmatrix}, \begin{bmatrix} -2 \\ 7 \\ 3 \\ 9 \end{bmatrix}, \begin{bmatrix} 6 \\ -4 \\ -7 \\ -5 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ -5 \\ 11 \end{bmatrix} \right\} \right\rangle \subseteq \mathbb{C}^4$

Typo -2

Basis of W? Use theorem BRS. Make vectors rows of a matrix, RREF

$$\xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$$

Theorem BRS $\Rightarrow$ B is a basis for W

-2

$$\underset{\sim}{x} = 3 \begin{bmatrix} -4 \\ \boxed{-3} \\ 3 \\ -11 \end{bmatrix} + 1 \begin{bmatrix} -2 \\ 7 \\ 3 \\ 9 \end{bmatrix} + 2 \begin{bmatrix} 6 \\ -4 \\ -7 \\ 5 \end{bmatrix} + -2 \begin{bmatrix} -1 \\ 0 \\ 3 \\ -4 \end{bmatrix} + 0 \begin{bmatrix} -4 \\ 5 \\ -5 \\ 11 \end{bmatrix} = \begin{bmatrix} 0 \\ \boxed{-10} \\ -8 \\ -6 \end{bmatrix} \in W$$

-7

$$\underset{\sim}{x} = 0 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \boxed{-10} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} + -8 \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -7 \\ -8 \\ -6 \end{bmatrix}$$

-7 1 6

Monday's Example Corrected

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Math 290 A, Tuesday March 31 Section D

Exam M (Wednesday, 10 AM)

Thu- PD (RQ)

Fri- Problem Session

Mon- DM (RQ) / Writing VS

<u>HW</u> $A = \begin{bmatrix} 1 & 2 & -1 & -6 \\ 0 & 1 & -2 & -2 \\ 1 & 1 & 1 & -4 \end{bmatrix}$, $N(A)$, <u>BNS</u> to get a basis

<u>Now</u> build a new basis, <u>different</u> from everybody else's

For <u>Thursday</u>

Theorem SSLD V - vector space, $S = \{\underset{\sim}{v_1}, \underset{\sim}{v_2}, \ldots, \underset{\sim}{v_t}\}$

S spans $V \Rightarrow$ <u>any</u> set of $t+1$ vectors in V is linearly dependent.

<u>Proof</u> $R = \{\underset{\sim}{u_1}, \underset{\sim}{u_2}, \ldots, \underset{\sim}{u_m}\} \subseteq V$, $m > t$.

$$\underset{\sim}{u_1} = a_{11}\underset{\sim}{v_1} + a_{21}\underset{\sim}{v_2} + \cdots + a_{t1}\underset{\sim}{v_t}$$

$$\underset{\sim}{u_2} = a_{12}\underset{\sim}{v_1} + a_{22}\underset{\sim}{v_2} + \cdots + a_{t2}\underset{\sim}{v_t}$$

$$\vdots$$

$$\underset{\sim}{u_m} = a_{1m}\underset{\sim}{v_1} + a_{2m}\underset{\sim}{v_2} + \cdots + a_{tm}\underset{\sim}{v_t}$$

Form a system of equations

$$\boxed{a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m} = 0$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2m}x_m = 0$$

$$\vdots$$

$$a_{t1}x_1 + a_{t2}x_2 + \cdots + a_{tm}x_m = 0$$

m variables, t equations, homogeneous

HMVEI $\Rightarrow$ infinitely many solutions

$R = \{\underset{\sim}{u_j} \mid 1 \le j \le m\}$

$$\underset{\sim}{u_j} = \sum_{i=1}^{t} a_{ij}\underset{\sim}{v_i} \quad 1 \le j \le m$$

$$\sum_{j=1}^{m} a_{ij}x_j = 0 \quad 1 \le i \le t$$

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$= 0\underset{\sim}{v_1} + 0\underset{\sim}{v_2} + \dots + 0\underset{\sim}{v_t} = \underset{\sim}{0} + \underset{\sim}{0} + \dots + \underset{\sim}{0} = \underset{\sim}{0}$

So we have a non-trivial relation of linear dependence on R

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(out of order)

Grab a non-trivial solution

$x_1 = c_1, \ x_2 = c_2, \ \ldots \ x_m = c_m$, not all

c_i zero.

Form

$$c_1 \underset{\sim}{u}_1 + c_2 \underset{\sim}{u}_2 + \cdots + c_m \underset{\sim}{u}_m$$

$$= c_1 (a_{11} \underset{\sim}{v}_1 + a_{21} \underset{\sim}{v}_2 + \cdots + a_{t1} \underset{\sim}{v}_t) +$$

$$c_2 (a_{12} \underset{\sim}{v}_1 + a_{22} \underset{\sim}{v}_2 + \cdots + a_{t2} \underset{\sim}{v}_t) +$$

$$\vdots$$

$$c_m (a_{1m} \underset{\sim}{v}_1 + a_{2m} \underset{\sim}{v}_2 + \cdots + a_{tm} \underset{\sim}{v}_t)$$

$$= (a_{11} c_1 + a_{12} c_2 + \cdots + a_{1m} c_m) \underset{\sim}{v}_1$$

$$+ (a_{21} c_1 + a_{22} c_2 + \cdots + a_{2m} c_m) \underset{\sim}{v}_2$$

$$\vdots$$

$$+ (a_{t1} c_1 + a_{t2} c_2 + \cdots + a_{tm} c_m) \underset{\sim}{v}_t$$

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$$x_j = c_j \qquad 1 \le j \le m$$

$$\sum_{j=1}^{m} c_j \underset{\sim}{u}_j = \sum_{j=1}^{m} c_j \sum_{i=1}^{t} a_{ij} \underset{\sim}{v}_i = \sum_{j=1}^{m} \sum_{i=1}^{t} c_j a_{ij} \underset{\sim}{v}_i$$

$$= \sum_{i=1}^{t} \sum_{j=1}^{m} c_j a_{ij} \underset{\sim}{v}_i$$

$$= \sum_{i=1}^{t} \left(\sum_{j=1}^{m} c_j a_{ij} \right) \underset{\sim}{v}_i$$

$$= \sum_{i=1}^{t} 0 \ \underset{\sim}{v}_i = \underset{\sim}{0}$$