Math 290 A, Thursday, April 2 Section PD

Fri – Section DM RQ

Mon – Problem Session
Writing VS

<u>Defn</u> V - vector space, $B = \{\underset{\sim}{v_1}, \underset{\sim}{v_2}, \ldots, \underset{\sim}{v_m}\}$ basis, then the dimension of V is m. $\dim(V) = m$ (\dim).

"<u>well-defined</u>"?

Ex $A = \begin{bmatrix} 1 & 2 & -1 & -6 \\ 0 & 1 & -2 & -2 \\ 1 & 1 & 1 & -4 \end{bmatrix}$ N(A)? $\xrightarrow{\text{RREF}}$ $\begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Basis from Theorem BNS $B = \left\{ \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$ Others

$\begin{bmatrix} -69\,000\,000 \\ 46\,000\,000 \\ 23\,000\,000 \\ 0 \end{bmatrix}, \begin{bmatrix} 420{,}000 \\ 420{,}000 \\ 0 \\ 210{,}000 \end{bmatrix}$ Sam W. / $\begin{bmatrix} 2/100 \\ 2/100 \\ 0 \\ 1/100 \end{bmatrix}, \begin{bmatrix} -300 \\ 200 \\ 100 \\ 0 \end{bmatrix}$ Ivan T / $\begin{bmatrix} -96 \\ 64 \\ 32 \\ 0 \end{bmatrix}, \begin{bmatrix} 64 \\ 64 \\ 0 \\ 32 \end{bmatrix}$ Sawyer H

$\begin{bmatrix} -12 \\ 8 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 0 \\ 3 \end{bmatrix}$ Rowan S-I / $\begin{bmatrix} -21 \\ 14 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ Jack R. / $\begin{bmatrix} -3\pi \\ 2\pi \\ \pi \\ 0 \end{bmatrix}, \begin{bmatrix} 2\pi \\ 2\pi \\ 0 \\ \pi \end{bmatrix}$ Sophia M.

$\left\{ 4\begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} + 6\begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$

Every basis has 2 vectors in it

$\dim(N(A)) = 2$

<u>Theorem</u> BIS V - vector space, B & C are bases, then $|B|=|C|$

<u>Proof</u> Suppose $|B|>|C|$ C spans V, so SSLD $\Rightarrow$ B linearly dependent $\not\Leftarrow$

Suppose $|C|>|B|$ B spans V so SSLD $\Rightarrow$ C linearly dependent $\not\Leftarrow$

So all that is left is $|B|=|C|$ depends on HMVEI

<u>Rank & Nullity</u>

$r = \dim(C(A))$ "rank"

$$\dim(C(A)) = \dim(C(A^t)) = \dim(R(A))$$

$$\text{rank } A = \text{rank } A^t$$

Section PD

Theorem G V vector space, $\dim(V) = m$

1) C, $|C| > m \Rightarrow C$ is linearly dependent
2) C, $|C| < m \Rightarrow C$ does not span V
3) $|C| = m$ + linearly independent set $\Longrightarrow C$ spans V
4) $|C| = m$ + C spans $V \Longrightarrow C$ linearly independent

Ex M_{22} $\quad X = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \, a + 2b - c + 3d = 0 \right\}$ subspace

$\dim(M_{22}) = 2 \cdot 2 = 4$

Spanning set

$$= \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \, a = -2b + c - 3d \right\} = \left\{ \begin{bmatrix} -2b + c - 3d & b \\ c & d \end{bmatrix} \middle| \, b, c, d \in \mathbb{C} \right\}$$

$$= \left\{ b \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix} + c \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + d \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix} \middle| \, b, c, d \in \mathbb{C} \right\} = \left\langle \underbrace{\left\{ \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix} \right\}}_{\text{spanning set} = S} \right\rangle$$

Also S is linearly independent. Use green "pattern of zeros & ones".

<u>Now</u> S' is a basis of X & $\dim(S') = 3$.

New set $R = \left\{ \begin{bmatrix} -7 & -2 \\ 1 & 4 \end{bmatrix}, \begin{bmatrix} -18 & 3 \\ 3 & 5 \end{bmatrix}, \begin{bmatrix} -19 & 4 \\ 1 & 4 \end{bmatrix} \right\} \subset X$ <u>Claim</u> R is <u>also</u> a basis.

(Check that $-7 + 2(-2) - (1) + 3(4) = 0$, ...)

Verify linear independence

$$a_1 \begin{bmatrix} -7 & -2 \\ 1 & 4 \end{bmatrix} + a_2 \begin{bmatrix} -18 & 3 \\ 3 & 5 \end{bmatrix} + a_3 \begin{bmatrix} -19 & 4 \\ 1 & 4 \end{bmatrix} = \underset{\sim}{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} -7a_1 - 18a_2 - 19a_3 &= 0 \\ -2a_1 + 3a_2 + 4a_3 &= 0 \\ a_1 + 3a_2 + a_3 &= 0 \\ 4a_1 + 5a_2 + 4a_3 &= 0 \end{aligned} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

4 equations, 3 variables
homogeneous

$\Rightarrow a_1 = a_2 = a_3 = 0$
So R is linearly independent.
By Theorem G, R <u>also</u> spans X.