Math 290A, Thursday, April 9   Section EE

Thu – EE (Sage)

Fri – PEE

Mon – SD (Sage)

Tue – Problems writing D&E

Wed – Exam D&E

$$A\underset{\sim}{x} = \lambda \underset{\sim}{x}$$

matrix vector scalar vector

defining equation eigenvalues & eigenvectors

MVP $\underset{\sim}{x} \neq \underset{\sim}{0}$

λ = eigenvalue

$\underset{\sim}{x}$ = eigenvector

Diagonalization

$A\underset{\sim}{x} = \lambda \underset{\sim}{x} \qquad \underset{\sim}{x} \neq \underset{\sim}{0}$

$A\underset{\sim}{x} - \lambda \underset{\sim}{x} = \underset{\sim}{0}$

$A\underset{\sim}{x} - \lambda I \underset{\sim}{x} = \underset{\sim}{0}$

$(A - \lambda I)\underset{\sim}{x} = \underset{\sim}{0} \qquad \underset{\sim}{x} \neq \underset{\sim}{0}$

$A - \lambda I$ singular

SMZD

$\underline{\det(A - \lambda I)} = 0$

$\downarrow$

characteristic polynomial

$\underset{\sim}{x} \in \underline{N(A - \lambda I)}$

eigenspace

all nonzero vectors

eigenvectors

Ex $A = \begin{bmatrix} 8 & -6 & 6 \\ 6 & -4 & 6 \\ -3 & 3 & -1 \end{bmatrix}$ eigen-stuff? $\det(\lambda I - A)$

$$\det(A - \lambda I_3) = \begin{vmatrix} 8-\lambda & -6 & 6 \\ 6 & -4-\lambda & 6 \\ -3 & 3 & -1-\lambda \end{vmatrix} = -\lambda^3 + 3\lambda^2 - 4$$

$$= -(\lambda-2)^2(\lambda+1) = p_A(\lambda)$$

eigenvalues

$\lambda = 2$ $\quad \alpha_A(2) = 2$

$\lambda = -1$ $\quad \alpha_A(-1) = 1$

eigenvectors (eigenspaces)

$$A - 2I_3 = \begin{bmatrix} 6 & -6 & 6 \\ 6 & -6 & 6 \\ -3 & 3 & -3 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

↑ singular $\qquad$ ↑ not I_3

$$\mathcal{E}_A(2) = N(A - 2I_3) = \left\langle \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\} \right\rangle$$

$\gamma_A(2) = 2$ ↩ dim $= 2$

$$A-(-1)I_3 = A+1I_3 = \begin{bmatrix} 9 & -6 & 6 \\ 6 & -3 & 6 \\ -3 & 3 & 0 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{E}_A(-1) = N(A+I_3) = \left\langle \left\{ \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix} \right\} \right\rangle \qquad \gamma_A(-1) = 1 \qquad \dim = 1$$