Math 290A, Tuesday, April 14, Problem Session

Wed - Exam D & E

Thu - LT (RQ)

Fri - $\overline{\text{ILT}}$

W/WF ; CR/NC

Sage / Exam

A.det( ) (determinant)

A.fcp()

(RREF)

EE.T20 A matrix, λ & ρ are eigenvalues, $\lambda \neq \rho$

$\Rightarrow \quad \mathcal{E}_A(\lambda) \cap \mathcal{E}_A(\rho) = \{\underset{\sim}{0}\}$ "eigenspaces are different"

<u>Proof</u> Suppose $\underset{\sim}{x} \in \mathcal{E}_A(\lambda) \cap \mathcal{E}_A(\rho)$, with ~~$\underset{\sim}{x} \neq \underset{\sim}{0}$~~.

$\underset{\sim}{x} \in \mathcal{E}_A(\lambda) \Rightarrow A\underset{\sim}{x} = \lambda \underset{\sim}{x}$

$\underset{\sim}{x} \in \mathcal{E}_A(\rho) \Rightarrow A\underset{\sim}{x} = \rho \underset{\sim}{x}$

$\Rightarrow \lambda \underset{\sim}{x} = \rho \underset{\sim}{x}$

$\Rightarrow \lambda \underset{\sim}{x} - \rho \underset{\sim}{x} = \underset{\sim}{0}$

$(\lambda - \rho)\underset{\sim}{x} = \underset{\sim}{0}$

SMEZV $\Rightarrow$ $\boxed{\lambda - \rho = 0 \quad \lambda = \rho \quad \cancel{\Leftrightarrow}}$ or $\underset{\sim}{x} = \underset{\sim}{0}$ $\cancel{\Leftrightarrow}$

① $\{\underset{\sim}{0}\} \subseteq \mathcal{E}_A(\lambda) \cap \mathcal{E}_A(\rho)$

$\underset{\sim}{0} \in \mathcal{E}_A(\lambda) \cap \mathcal{E}_A(\rho)$

② $\mathcal{E}_A(\lambda) \cap \mathcal{E}_A(\rho) \subseteq \{\underset{\sim}{0}\}$

Grab $\underset{\sim}{y} \in \mathcal{E}_A(\lambda) \cap \mathcal{E}_A(\rho)$

$\Rightarrow \underset{\sim}{y} = \underset{\sim}{0} \Rightarrow \underset{\sim}{y} \in \{\underset{\sim}{0}\}$

Theorem SMEZV

If $\alpha \underset{\sim}{u} = \underset{\sim}{0} \Rightarrow \alpha = 0$ or $\underset{\sim}{u} = \underset{\sim}{0}$

①+② Defn SE