Math 290A, Thursday, April 16, Section LT

$T: \underset{\sim}{U} \to V$ T function, U & V vector spaces

1) $T(\underset{\sim}{u_1} + \underset{\sim}{u_2}) = T(\underset{\sim}{u_1}) + T(\underset{\sim}{u_2})$

2) $T(\alpha \underset{\sim}{u}) = \alpha T(\underset{\sim}{u})$

elements of V

Ex $T: \mathbb{C}^4 \to \mathbb{C}^2$ $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 + 3x_2 - 6x_3 + x_4 \\ 9x_1 + 2x_3 - 5x_4 \end{bmatrix}$

verify $T(\underset{\sim}{x_1} + \underset{\sim}{x_2}) = T(\underset{\sim}{x_1}) + T(\underset{\sim}{x_2})$
$T(\alpha \underset{\sim}{x}) = \alpha T(\underset{\sim}{x})$] algebra (like closure, not closure)

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 \\ 9x_1 \end{bmatrix} + \begin{bmatrix} 3x_2 \\ 0x_2 \end{bmatrix} + \begin{bmatrix} -6x_3 \\ 2x_3 \end{bmatrix} + \begin{bmatrix} x_4 \\ -5x_4 \end{bmatrix} = x_1 \begin{bmatrix} 2 \\ 9 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -6 \\ 2 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ -5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & -6 & 1 \\ 9 & 0 & 2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Ex $T: \mathbb{C}^3 \to \mathbb{C}^2$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Prove $T(\underset{\sim}{x}) = A\underset{\sim}{x}$ is a linear transformation for any matrix A.

Fact LTTZZ

$$T: U \to V \Rightarrow T(\underset{\sim}{0}_U) = \underset{\sim}{0}_V$$

Proof

$$\begin{aligned} \underset{\sim}{0} &= T(\underset{\sim}{0}) - T(\underset{\sim}{0}) \\ &= T(\underset{\sim}{0} + \underset{\sim}{0}) - T(\underset{\sim}{0}) \\ &= T(\underset{\sim}{0}) + \underline{T(\underset{\sim}{0}) - T(\underset{\sim}{0})} \\ &= T(\underset{\sim}{0}) + \underset{\sim}{0} \\ &= T(\underset{\sim}{0}) \end{aligned}$$

$$
\begin{tikzcd}
u_1, u_2 \arrow[r, "T"] \arrow[d, "+_U"'] & T(\underset{\sim}{u_1}), T(\underset{\sim}{u_2}) \arrow[d, "+_V"] \\
\underset{\sim}{u_1} + \underset{\sim}{u_2} \arrow[r, "T"'] &
\end{tikzcd}
$$

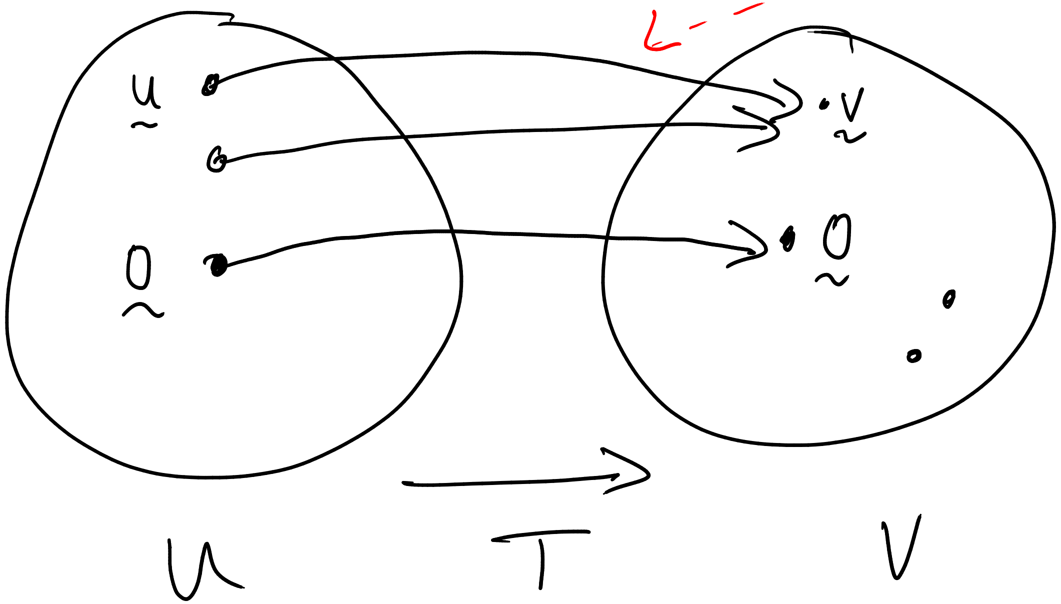
$T(\underset{\sim}{u_1}) + T(\underset{\sim}{u_2})$

$T(\underset{\sim}{u_1} + \underset{\sim}{u_2})$

equal if T is a l.t.

$\rightarrow T(\underset{\sim}{u}) = \underset{\sim}{v}$

Picture: $T(\alpha \underset{\sim}{u}) = \alpha T(\underset{\sim}{u})$



Theorem LTDB

"It is enough to know what a linear transformation does to a basis"