Math 290 A, Monday, April 20 Section SLT

Email: Read-only

Mon: SLT

Tue: Problem Session

Wed: !

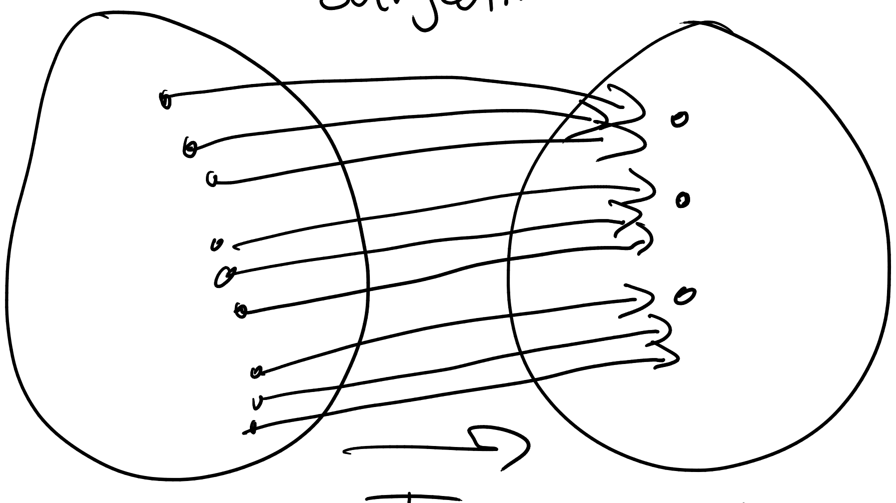
Thu: IVLT

Fri: Problem Session

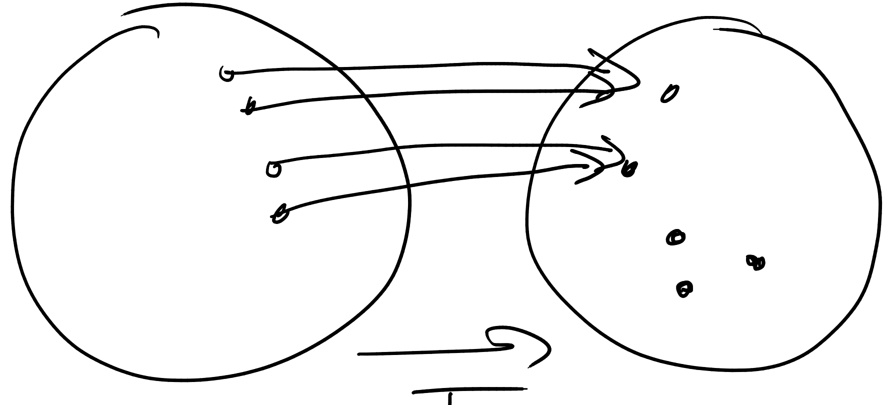
Mon: VR (Sage) Writing

__Defn__ $T : U \to V$ is __surjective__ if for each $\underset{\sim}{v} \in V$ there is a $\underset{\sim}{u} \in U$ so that $T(\underset{\sim}{u}) = \underset{\sim}{v}$.

surjective



non-surjective



<u>Ex</u> $T: P_2 \to P_3$

$T(a+bx+cx^2) = (-5a+4b-6c) + (-6a+5b-7c)x + (-a+b-c)x^2 + (3a-2b+4c)x^3$

Find $a+bx+cx^2$ so $T(a+bx+cx^2) = 2-x-3x^2+4x^3$

Grab $2-x-3x^2+4x^3 \in P_3$

$(-5a+4b-6c) + (-6a+5b-7c)x + (-a+b-c)x^2 + (3a-2b+4c)x^3 = 2-x-3x^2+4x^3$.

system:
$$\begin{aligned} -5a+4b-6c &= 2 \\ &= -1 \\ &= \\ 3a-2b+4c &= 4 \end{aligned}$$

4 equations, 3 variables

<u>not</u> homogenous $\longrightarrow$ RREF augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

RCLS $\Rightarrow$ no solution.

So there is no ax^2+bx+c

$T^{-1}(2-x-3x^2+4x^3) = \emptyset$

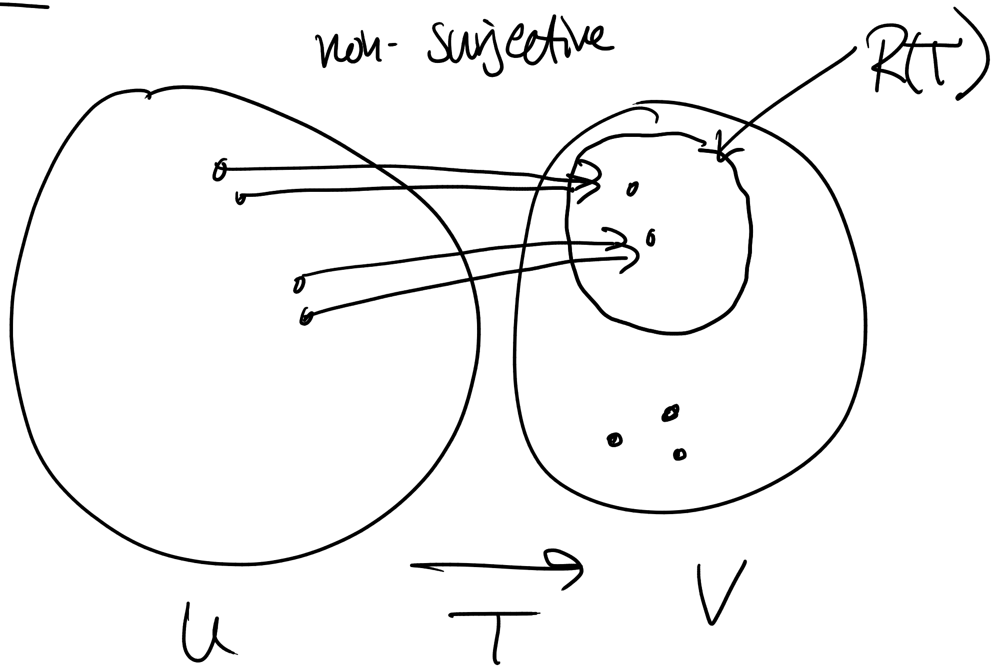
Defn The <u>range</u> of $T: U \to V$ is

$$R(T) = \{ T(\underset{\sim}{u}) \mid \underset{\sim}{u} \in U \} \subseteq V$$

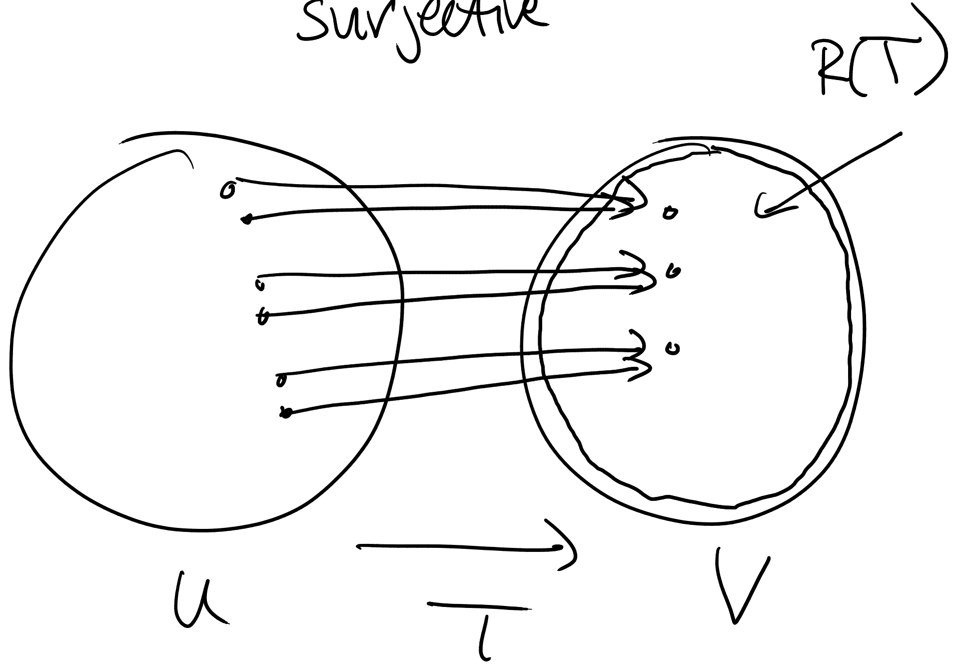
Fact $R(T)$ is a subspace of V.

Theorem RSLT $\quad T$ surjective $\iff R(T) = V$

non-surjective



surjective



__Theorem__ SSRLT Suppose $T: U \to V$ & $B = \{\underset{\sim}{u_1}, \underset{\sim}{u_2}, \dots, \underset{\sim}{u_p}\}$ ← basis?
is a spanning set for U. Then $C = \{T(\underset{\sim}{u_1}), T(\underset{\sim}{u_2}), \dots, T(\underset{\sim}{u_p})\}$
is a spanning set for $R(T)$.

__Ex__ $T: M_{22} \to P_2$ $\quad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+b+c+4d) + (-a+2c-5d)x + (2a+3b+6c+6d)x^2$

$R(T)$?

Spanning set for M_{22} (basis!) $= \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\} = B$

$C = \left\{T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right), T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right), T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right), T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right)\right\}$

$= \{1 - x + 2x^2, 1 + 3x^2, 1 + 2x + 6x^2, 4 - 5x + 6x^2\}$

So $R(T) = \langle C \rangle$

C is linearly dependent.

Slim down C to a linearly independent set.

$\alpha_1(1 - x + 2x^2) + \alpha_2(1 + 3x^2) + \alpha_3(1 + 2x + 6x^2) + \alpha_4(4 - 5x + 6x^2) = \underset{\frown}{0} = 0 + 0x + 0x^2$

homogeneous system w/ four variables, three equations

coefficient matrix $\xrightarrow{\text{RREF}}$ $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

Solution
$\alpha_4 = 1$
$\alpha_1 = -3$
$\alpha_2 = -2$
$\alpha_3 = 1$

So $1(4-5x+6x^2) = 3(1-x+2x^2) + 2(1+3x^2) + (-1)(1+2x+6x^2)$

$$R(T) = \langle C \rangle = \langle \underbrace{\{1-x+2x^2,\ 1+3x^2,\ 1+2x+6x^2\}} \rangle$$

linearly independant (so basis)

So $\dim(R(T)) = \boxed{3}$

& $\dim P_2 = \boxed{3}$

$\underline{R(T)} \subseteq \underline{P_2}$

$\Longrightarrow R(T) = P_2 \Rightarrow T$ is a surjective L.T.