Math 290A, Friday, April 24 Problem Session

Mon - VR (RQ)
Writing 10AM

Thu - MR

Wed - Exam LT

U & V vector spaces then can say U & V are <u>isomorphic</u>. Write $U \cong V$

An <u>isomorphism</u> is an invertible linear transformation.

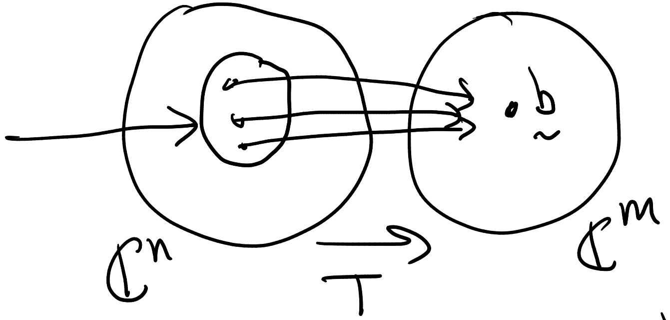
$LS(A, \underset{\sim}{b})$ system of linear equations ($A\underset{\sim}{x} = \underset{\sim}{b}$ SLEMM)

LT: $T : \mathbb{C}^n \to \mathbb{C}^m$ $\quad T(\underset{\sim}{x}) = A\underset{\sim}{x}$

Solutions $T^{-1}(\underset{\sim}{b})$

solution set

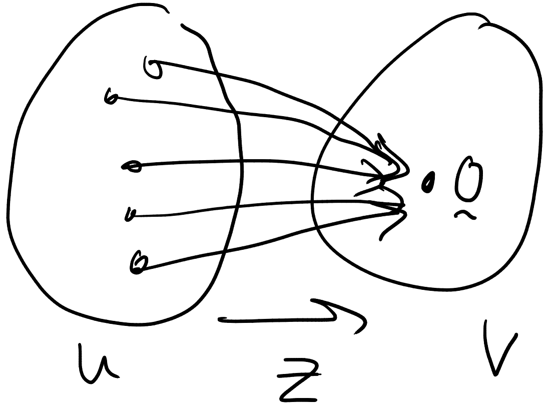
$T^{-1}(\underset{\sim}{b})$



Solutions to $LS(A, \underset{\sim}{0}) = N(A) = T^{-1}(\underset{\sim}{0}) = K(T)$

homogeneous

<u>ILT.M60</u> $Z: U \to V$ $Z(\mathbf{u}) = \mathbf{0}_V$



When is Z injective? (Condition on U.)

Answer: $U = \{\mathbf{0}\}$

<u>Fact</u> Z injective $\iff U = \{\mathbf{0}\}$

<u>Proof</u> $(\Leftarrow)$ $K(Z) = \{\mathbf{u} \in U \mid Z(\mathbf{u}) = \mathbf{0}\}$ Defn of kernel

$= U$ Defn of Z

$= \{\mathbf{0}\}$ Hypothesis

Then Theorem KILT says Z injective.

$(\Rightarrow)$ Z injective $\Rightarrow K(Z) = \{\mathbf{0}\}$ KILT

But $K(Z) = U$ $\longleftrightarrow$ $U = K(Z) = \{\mathbf{0}\}$

IVLT.T40

$T: U \to V$ $\quad$ $\dim(U) = \dim(V) = \dim(W)$ $\quad$ S invertible

$S: V \to W$ $\quad$ $S \circ T$ invertible $\quad \Rightarrow$ T invertible

$K(T) \subseteq K(S \circ T) = \{\underset{\sim}{0}\}$ (ILT.T15) $\qquad \underset{\sim}{u} \in K(T)$, $(S \circ T)(\underset{\sim}{u}) = S(T(\underset{\sim}{u}))$

$= S(\underset{\sim}{0}) = \underset{\sim}{0}$

$\Rightarrow \underset{\sim}{u} \in K(S \circ T)$

$r(T) + n(T) = \dim(\text{domain})$

$$\begin{aligned} r(T) &= \underbrace{r(T) + \overbrace{n(T)}^{\text{zero}}}\, - n(T) \\ &= \dim(U) - n(T) \\ &= \dim(U) - 0 \\ &= \dim(U) \\ &= \dim(V) \Rightarrow R(T) = V \Rightarrow T \text{ surjective} \end{aligned}$$

$(K(T) \subseteq \{\underset{\sim}{0}\} \Rightarrow K(T) = \{\underset{\sim}{0}\} \Rightarrow n(T) = 0)$

T injective

T invertible

$R(S \circ T) \subseteq R(S)$ (SLT.T15)