Math 290A, Monday, April 27 Section VR

Mon - VR Sage
- writing

Tue - MR (RQ)

Wed - Exam LT

"R" - representation

Defn V - vector space, $\underset{\sim}{v} \in V$,
B basis of V, $B = \{\underset{\sim}{u}_1, \underset{\sim}{u}_2, \ldots, \underset{\sim}{u}_n\}$. Then
$\underset{\sim}{v} = a_1 \underset{\sim}{u}_1 + a_2 \underset{\sim}{u}_2 + \cdots + a_n \underset{\sim}{u}_n$ (by Thm VRRB).
Then
$$\rho_B(\underset{\sim}{v}) = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

Fact $\rho_B : V \to \mathbb{C}^n$ is a bijective linear transformation.

"bijection" = injective + surjective

Ex In $\mathbb{C}^3$, $\underset{\sim}{v} = \begin{bmatrix} -12 \\ 9 \\ 16 \end{bmatrix}$, $B = \left\{ \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix} \right\}$

$$\rho_B(\underset{\sim}{v}) = \rho_B\left(\begin{bmatrix} -12 \\ 9 \\ 16 \end{bmatrix}\right) = \rho_B\left(2\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} + (-1)\begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix} + 2\begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

Different basis $C = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

$$\rho_C(\underset{\sim}{v}) = \rho_C\left(\begin{bmatrix} -12 \\ 9 \\ 16 \end{bmatrix}\right) = \rho_C\left((-12)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 9\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 16\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -12 \\ 9 \\ 16 \end{bmatrix}$$

Ex M_{22} $\underset{\sim}{w} = \begin{bmatrix} 8 & 7 \\ 23 & 21 \end{bmatrix}$ $B = \left\{ \begin{bmatrix} -2 & 4 \\ 3 & 3 \end{bmatrix}, \begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 5 \\ 3 & 5 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 5 & 5 \end{bmatrix} \right\}$

$$\rho_B\left(\begin{bmatrix} 8 & 7 \\ 23 & 21 \end{bmatrix}\right) = \rho_B\left(2\begin{bmatrix} -2 & 4 \\ 3 & 3 \end{bmatrix} + 0\begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix} + (-1)\begin{bmatrix} 0 & 5 \\ 3 & 5 \end{bmatrix} + 4\begin{bmatrix} 3 & 1 \\ 5 & 5 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 4 \end{bmatrix}$$

$D = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ $\quad \rho_D\left(\begin{bmatrix} 8 & 7 \\ 23 & 21 \end{bmatrix} \right) = \begin{bmatrix} 8 \\ 7 \\ 23 \\ 21 \end{bmatrix}$

$\uparrow$

Ex $\rho_B : M_{22} \to \mathbb{C}^4$ invertible, $\rho_B^{-1} : \mathbb{C}^4 \to M_{22}$

$$\rho_B^{-1}\left(\begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right) = 1 \begin{bmatrix} -2 & 4 \\ 3 & 3 \end{bmatrix} + (-2) \begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix} + 0 \begin{bmatrix} 0 & 5 \\ 3 & 5 \end{bmatrix} + 1 \begin{bmatrix} 3 & 1 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 9 \\ 2 & 10 \end{bmatrix}$$

Theorem Suppose $\dim(U) = \dim(V)$ then U & V are isomorphic

Ex $\dim(P_{12}) = 13$ & $\dim(\mathbb{C}^{13}) = 13 \Rightarrow P_{12} \cong \mathbb{C}^{13}$

Coordinatization Principle

(Section MR)

Defn $T: U \to V$ $B = \{\underset{\sim}{u_1}, \underset{\sim}{u_2}, \dots, \underset{\sim}{u_n}\}$ basis of U, C basis of V. $A\underset{\sim}{x} = \lambda \underset{\sim}{x}$

The matrix representation of T, relative to B & C is

$$M^T_{B,C} = \left[\rho_C(T(\underset{\sim}{u_1})) \,\middle|\, \rho_C(T(\underset{\sim}{u_2})) \,\middle|\, \cdots \,\middle|\, \rho_C(T(\underset{\sim}{u_n})) \right]$$

Ex $T: P_2 \to P_2$ $T(a+bx+cx^2) = (5a+2b+2c) + (-10a-3b-2c)x + (6a+2b+c)x^2$

$B = C = \{1-2x+x^2, 1-3x+x^2, x-x^2\}$

$$\rho_C(T(\underset{\sim}{u_1})) = \rho_C(3-6x+3x^2) = \rho_C(3(1-2x+x^2)+0(1-3x+x^2)+0(x-x^2)) = \begin{bmatrix}3\\0\\0\end{bmatrix}$$

$$\rho_C(T(\underset{\sim}{u_2})) = \rho_C(1-3x+x^2) = \rho_C(0(1-2x+x^2)+1(1-3x+x^2)+0(x-x^2)) = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$\rho_C(T(\underset{\sim}{u_3})) = \rho_C(-x+x^2) = \rho_C(0(1-2x+x^2)+0(1-3x+x^2)+(-1)(x-x^2)) = \begin{bmatrix}0\\0\\-1\end{bmatrix}$$

$$M^T_{B,C} = \begin{bmatrix}3&0&0\\0&1&0\\0&0&-1\end{bmatrix}$$