Math 290 Group Office Hours Friday, May 8

MINM. T11

A Hermitian $\Rightarrow$ $[A]_{ii} \in \mathbb{R}$

"diagonal entry" ↑ reals (not $\mathbb{C}$)

<u>Proof</u>

Know $A^* = A$; $(\overline{A})^t = A$

$$[A]_{ii} = [(\overline{A})^t]_{ii}$$

$$= [\overline{A}]_{ii} \qquad \text{swap } i \text{ w/ } i$$

$$= \overline{[A]_{ii}} \qquad \underline{\text{Defn}} \text{ CCM}$$

$$\alpha = \overline{\alpha} \iff \alpha \text{ real}$$

MINM. T12

$U_{n \times n}$ U unitary? $UU^* = I$?

CUMOS: columns of U orthonormal set ?

$\underset{\sim}{u}_i$

Each inner product is n multiplications ($n-1$ additions)

$$\langle \underset{\sim}{u}_i, \underset{\sim}{u}_j \rangle = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$$= \overline{\langle \underset{\sim}{u}_j, \underset{\sim}{u}_i \rangle}$$

$\underset{i}{n} \cdot \underset{j}{n} = n^2$

Use IPAC

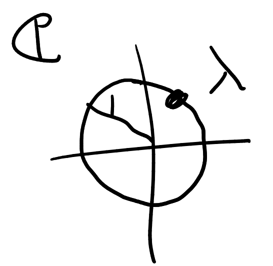
$\underset{i \neq j}{n(n-1)/2} + \underset{i\,i}{n} = \frac{n(n-1)}{2} + \frac{2n}{2} = \frac{n^2-n+2n}{2}$

$= \frac{n^2+n}{2} = \frac{n(n+1)}{2}$

$z = a + bi \qquad \|z\| = \sqrt{a^2+b^2} = \sqrt{z\bar{z}} \qquad \|xy\| = \|x\|\,\|y\|$

PIE. T22

U unitary $\quad \lambda$ eigenvalue $\Rightarrow |\lambda| = 1$



$\mathbb{R}$

Proof

Grab eigenvector $\underset{\sim}{x}$ $(\underset{\sim}{x} \neq \underset{\sim}{0})$

$\frac{1}{\langle \underset{\sim}{x}, \underset{\sim}{x} \rangle}$

① $\langle U\underset{\sim}{x}, U\underset{\sim}{x} \rangle = \langle \lambda \underset{\sim}{x}, \lambda \underset{\sim}{x} \rangle = \bar{\lambda} \langle \underset{\sim}{x}, \lambda \underset{\sim}{x} \rangle$

$= \bar{\lambda}\lambda \langle \underset{\sim}{x}, \underset{\sim}{x} \rangle$ ①

②

② $\langle U\underset{\sim}{x}, U\underset{\sim}{x} \rangle = \langle U^* U \underset{\sim}{x}, \underset{\sim}{x} \rangle = \langle I_n \underset{\sim}{x}, \underset{\sim}{x} \rangle = \langle \underset{\sim}{x}, \underset{\sim}{x} \rangle$

So ① ② $\Rightarrow \bar{\lambda}\lambda \cancel{\langle \underset{\sim}{x}, \underset{\sim}{x} \rangle} = \cancel{\langle \underset{\sim}{x}, \underset{\sim}{x} \rangle} \Rightarrow \bar{\lambda}\lambda = 1 \Rightarrow \sqrt{\bar{\lambda}\lambda} = 1$

$|\lambda| = 1$

$\underset{\sim}{x} \neq \underset{\sim}{0}$, PIP $\Rightarrow \langle \underset{\sim}{x}, \underset{\sim}{x} \rangle \neq 0$