Math 290 B Friday March 13

V - vector space, $S = \{\underset{\sim}{v}_1, \underset{\sim}{v}_2, \dots, \underset{\sim}{v}_m\}$

$\langle S \rangle$ = span of S = all linear combos of vectors of S

Ex P_3 all polys w/ degree 3 or less

$\underset{\sim}{v} = -3 - 2x + 4x^2 + x^3 \in \langle S \rangle$?

$S = \{1 - 2x + x^2 + 3x^3,\ 2 + 3x^2 + x^3\}$

$\underset{\sim}{v} = -3 - 2x + 4x^2 + x^3 = a(1 - 2x + x^2 + 3x^3) + b(2 + 3x^2 + x^3)$ $\underline{\underline{a, b?}}$

$= (a - 2ax + ax^2 + 3ax^3) + (2b + 3bx^2 + bx^3)$

$= (a + 2b) + (-2a)x + (a + 3b)x^2 + (3a + b)x^3$

$-3 = a + 2b$

$-2 = -2a$

$4 = a + 3b$

$1 = 3a + b$

augmented matrix $\xrightarrow{\text{RREF}}$ $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right]$

RCS $\Rightarrow$ no solution

no a, b

so $\underset{\sim}{v} \notin \langle S \rangle$

Ex M_{22} $\quad S = \left\{ \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \right\}$ S linearly independent?

$$a\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} + c\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} = \underset{\sim}{0}$$

$$\begin{bmatrix} 2a+b+2c & a+c \\ -a+3b+c & 2a+b-c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} 2a+b+2c &= 0 \\ a+c &= 0 \\ -a+3b+c &= 0 \\ 2a+b-c &= 0 \end{aligned}$$

~~augm~~ coeff matrix $\xrightarrow{\text{~~RREF~~}}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

The **only** solution is $a = b = c = 0$

This ~~needs~~ means that S **is** linearly independent

Theorem A $m \times n$ matrix, then $N(A)$ is a subspace of $\mathbb{C}^n$

Proof 1) $\underset{\sim}{0} \in N(A)$? $A\underset{\sim}{0} = \underset{\sim}{0}$ so $N(A) \neq \emptyset$.

is in $N(A)$

TSS

2) Suppose $\underset{\sim}{x}, \underset{\sim}{y} \in N(A)$. $A\underset{\sim}{x} = \underset{\sim}{0}$, $A\underset{\sim}{y} = \underset{\sim}{0}$ (SLEMM)

Consider $A(\underset{\sim}{x}+\underset{\sim}{y}) = A\underset{\sim}{x} + A\underset{\sim}{y} = \underset{\sim}{0} + \underset{\sim}{0} = \underset{\sim}{0}$ So, yes, $\underset{\sim}{x}+\underset{\sim}{y} \in N(A)$.

MMDAA

3) Suppose $\alpha \in \mathbb{C}$, $\underset{\sim}{x} \in N(A)$. Know $A\underset{\sim}{x} = \underset{\sim}{0}$.

Consider $A(\alpha\underset{\sim}{x}) = \alpha(A\underset{\sim}{x}) = \alpha \cdot \underset{\sim}{0} = \underset{\sim}{0}$ So, yes, $\alpha\underset{\sim}{x} \in N(A)$.

Ex $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \ 5x_1 - x_2 + 8x_3 = 0 \right\} = N([5\ -1\ 8])$

before

$\subseteq \mathbb{C}^3$

subspace

Theorem V - vector space, $S = \{\underset{\sim}{v}_1, \underset{\sim}{v}_2, \ldots, \underset{\sim}{v}_m\}$, $\langle S \rangle$ is a subspace of V

Proof TSS

1) $\langle S \rangle \neq \emptyset$? $\underset{\sim}{0} \in \langle S \rangle$? $\left[\alpha_1 \underset{\sim}{v}_1 + \alpha_2 \underset{\sim}{v}_2 + \cdots + \alpha_m v_m \overset{?}{=} \underset{\sim}{0}\right]$

Notice $0\underset{\sim}{v}_1 + 0\underset{\sim}{v}_2 + \cdots + 0\underset{\sim}{v}_m \in \langle S \rangle$

$\underset{\sim}{0} + \underset{\sim}{0} + \cdots + \underset{\sim}{0} \in \langle S \rangle$

$\underset{\sim}{0} \in \langle S \rangle$

2) Suppose $\underset{\sim}{x}, \underset{\sim}{y} \in \langle S \rangle$.

Know $\underset{\sim}{x} = \alpha_1 \underset{\sim}{v}_1 + \alpha_2 v_2 + \cdots + \alpha_m \underset{\sim}{v}_m$, $\underset{\sim}{y} = \beta_1 \underset{\sim}{v}_1 + \beta_2 \underset{\sim}{v}_2 + \cdots + \beta_m \underset{\sim}{v}_m$

Consider

$$\underset{\sim}{x} + \underset{\sim}{y} = (\alpha_1 \underset{\sim}{v}_1 + \cdots + \alpha_m v_m) + (\beta_1 \underset{\sim}{v}_1 + \cdots + \beta_m \underset{\sim}{v}_m)$$

$$= (\alpha_1 \underset{\sim}{v}_1 + \beta_1 \underset{\sim}{v}_1) + \cdots + \cancel{(\alpha_m (\beta_m) v_m)} (\alpha_m \underset{\sim}{v}_m + \beta_m \underset{\sim}{v}_m)$$

$$= \underbrace{(\alpha_1 + \beta_1)}_{\text{scalar}} \underset{\sim}{v}_1 + \cdots + (\alpha_m + \beta_m) \underset{\sim}{v}_m \in \langle S \rangle$$

3) HW