Math 290 B Thursday, March 26 Section LISS

V vector spaces, set (of vectors), vector addition, scalar multiplications

"10 good things" + theorems

Now have <u>linear combinations</u>

RQ

① $\langle R \rangle$ - subspace, <u>span</u>

② R spans P_2

<u>Ex</u> P_2 - all polys with degree 2 or less

$R = \{1-x+2x^2, 1+3x^2, 1-x+3x^2\} \subseteq P_2$

<u>Claim</u> "R spans P_2", "R is a spanning set for P_2", $\langle R \rangle = P_2$

Defn SE
① $\langle R \rangle \subseteq P_2$ (EZ)
② $P_2 \subseteq \langle R \rangle$

Grab $a+bx+cx^2 \in P_2$.

Is $a+bx+cx^2 = a_1(1-x+2x^2)+a_2(1+3x^2)+a_3(1-x+3x^2)$?

$$= (a_1+a_2+a_3)+(-a_1-a_3)x+(2a_1+3a_2+3a_3)x^2$$

$$\begin{aligned} a_1+a_2+a_3 &= a \\ -a_1 \quad -a_3 &= b \\ 2a_1+3a_2+3a_3 &= c \end{aligned} \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3a-c \\ 0 & 1 & 0 & a+b \\ 0 & 0 & 1 & -3a-b+c \end{array}\right]$$

(unique) solution NMUS for <u>any</u> choice of a, b & c

So $P_2 \subseteq \langle R \rangle$ & $P_2 = \langle R \rangle$

<u>Ex</u> $W = \{a+bx+cx^2 \mid a-4b+3c=0\}$ <u>is</u> a subspace of P_2

Find a spanning set of W.

$$W = \{a + bx + cx^2 \mid a = 4b - 3c\}$$

$[\,1\ \ -4\ \ 3\,]$

a dependent

b, c free

$a = 4b - 3c$

$$= \{(4b-3c) + bx + cx^2 \mid b, c \in \mathbb{C}\}$$

$$= \{(4b + bx) + (-3c + cx^2) \mid b, c \in \mathbb{C}\}$$

$$= \{b(4+x) + c(-3+x^2) \mid b, c \in \mathbb{C}\}$$

$$= \langle \{4+x,\ -3+x^2\} \rangle$$